UNIVERSITÉ DE YAOUNDÉ I
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THE UNIVERSITY OF YAOUNDÉ I
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FREGE AND THE LOGICAL REVOLUTION: A READING OF THE BEGRIFTSSCHRIFT

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To my elder sister

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ABSTRACT

"Frege logical revolution: a reading of the *Begriffsschrift"* is part of the vast movement of analytical philosophy and even classical logic or formal logic still here considered as the Aristotelian logic. By revolutionizing the classical logic, it signs up in the approach of modern logic also called mathematical logic. The aim here is to examine the epistemological relevance that might have the Ideography in relation to the Fregean conception of logic and the way in which this has been structured and constructed in order to advance logic.

The thesis is as follows: the logic of frege in the epistemological and socio-political plan is necessary in Africa today dive into dogmatism and mysticism. The African people should stop thinking that myth alone can be the source of solution to the present problems of their continent. It is only with logical and rational thinking that African can be able to think and bring solutions to their problems of under-development.

Key words: **Logic, Pre-Fregean, Ideography, revolution, classical logic and modern logic.**

RESUME

<<La révolution logique de Frege : une lecture Begriffsschrift>> s'inscrit dans le vaste mouvement de la philosophie analytique et même de la logique classique ou de la logique formelle jusqu'ici considérée comme la logique aristotélicienne. Il s'agit ici d'examiner la pertinence épistémologique que pourrait avoir l'idéographie relativement à la conception frégéenne de la logique para port à la manière dont elle est structurée et construite pour faire avancer la logique.

La thèse principale est la suivante : la logique frégéenne sur les plans épistémologique et socio-politique est nécessaire dans l'Afrique aujourd'hui plongée dans le dogmatisme et le mysticisme. Le peuple africain devrait cesser de penser que seul le mythe peut être la source des solutions aux problèmes actuels du continent. Ce n'est qu'avec la pensée logique et rationnelle que l'Afrique peut pouvoir penser et apporter des solutions à leurs problèmes de sous-développement.

Mot clés : ***logique, Pre-Frégienne, Idéographie, révolution, logique classique et logique moderne***.

GENERAL INTRODUCTION

"*We use the word logic frequently, so it is legitimate to ask us what notions are related to it. It's easy to set several.*"[1] From its original word "*logos*" in Greek, this signified discourse or reason. This is the discourse on the method and principle use to distinguish between correct from incorrect reasoning. It can also be considered as the systematic study of valid reasoning or the study of method for evaluating argument. "*for it is not enough having good reason (a process in which a conclusion is drawn from a set of premises) for the most important is to apply it well"*[2] This implies that though "*everything in nature, as well in the inanimate as in the animate word, happen or is done according to rules"*[3] and that every human has the ability to reason, but not everyone reason and apply this rules correctly. The correct reasoning and application of rules is what is known as logic. The *Vocabulaire Technique et Critique de la Philosophie* defined logic as :

> *The science's which investigate the general principle of valid thought. Its object is to discuss the characteristic of judgment regarded not as psychological phenomena, but as expressing our knowledge and belief; and in particular, it seeks to determine the condition under which we are justified in passing from a given judgment to another judgment that follow from them. It may accordingly be described as a normative science: the character it possesses in common with ethic and aesthetics*[4]

Looking at the view of some philosophers like Hegel, Kant, Frege and Aristotle considered as the father of logic, logic is "*the science of the law of reasoning"*[5]. Reason which can be taken here as the faculty which help us to construct good, coherence and rigorous judgment and distinguish between the bad and the good. Meaning that, logic is there as the sciences, which help us to carry out a critical reflection and suggest solution on the different rules or laws and prepositions guiding our thoughts.

This definition of logic can be seen again with Kant when he affirms: *"logic is the science of the rule of understanding in general.*"[6] He continues by saying:

> *A science of the necessary laws of thinking without which no use of the understanding or of reasoning has place, and which are by*

[1]Jean-Blaise Grize, *Logique Moderne,* Belgium, Mouton & Co, 1969, Introduction. « *Nous utilisons fréquemment le mot "logique", aussi est-il légitime de nous demander quelles notions y sont liées. Il est facile d'en dégager plusieurs.* »

[2] René Descartes, *Discourse on method (1637),* trans. en. Donald A. Cress, Cambridge, Hackett Publishing Company, 1998,

[3] Emmanuel Kant, *Logic,* trans. en. John Richardson, Ludgate-street, W. Simpkin & R. Marshall, Stationers Court, 1819, p. 09.

[4] André Lalande, *Vocabulaire Technique et Critique de la Philosophie*, Paris, PUF, 2010, pp. 572-573.

[5] Aristotle, The *Organon, or Logical Treaties, of Aristotle*, trans. en. Octavious Freire Owen, London, George Bell & Sons, 1889, introduction.

[6] Emmanuel Kant, *Critique of Pure Reason* (1781) trans. en. Paul Guyer, Cambridge, Cambridge University Press, 1998, p.194.

> *consequences the sole conditions, on which the understanding can agree with itself or be consistent, - the necessary laws and condition of its right use, - logic however is a Canon. A canon of the understanding and reason, must of course not borrow principles either from any science, or any experience whatever: it must comprehend nothing but laws a prior which are necessary appertain to the understanding in general.*[7]

Understanding here is considered as the reason or as Kant called it the intuition. The definition of logic as we have just seen shows us that logic is the epistemological study of thought or "Idea" as define Andre Leonard:

> *logic is the science of the idea in and for oneself, that is to say the science of pure idea, of pure intelligibility in the abstraction of its only though of oneself, the science of the idea still has neither the partiality of a natural in oneself nor that of the spiritual for oneself, but which remains in the impartiality of its in-and -for self, logic and thus has no other partiality than its very impartiality.*[8]

According to the mathematician, logician and philosopher Gottlob Frege, logic is "*a way that, disregarding the particular characteristics of an object, depend solely on those laws upon which all knowledge rests*."[9] This mean that "*logic is that science which is based on those laws of thought that transcendence all particulars.*"[10] Through this affirmation, our author shows that logic has to be objective in a sense that this does not take into consideration any subjective or contingency such as ordinary language and grammar. To him "*is logical what is thought or built outside of any intuition; is logical what is general to the point of belonging to any language and such that one cannot conceive of a language that is private.*"[11] It becomes therefore important for us to revolutionize logic for it to be precise and more determine for use to prevent ambiguity.

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[7] Emmanuel Kant, *Logic*, p. 13.

[8] André Leonard, *Commentaire Littéral de la logique de Hegel*, Paris, Librairie philosophique, J. Vrin, 1974, p.19. « *La logique est la science de l'Idée en et pour soi, c'est-à-dire la science de l'Idée pure, de la pure intelligibilité dans l'abstraction de sa seule pensée de soi, la science de l'Idée qui n'a encore ni la partialité d'un en-soi naturel ni celle d'un pour-soi spiritual, mais qui demeure dans l'impartialité de son en-et-pour soi logique et ainsi n'as d'autre partialité que son impartialité même.* »

[9] Gottlob Frege, Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought, New York, Lubrecht & Cramer, 1879, Preface.

[10] Id.

[11] This is our translation of : « *est logique ce qui est pensé ou construit en dehors de toute intuition ; est logique ce qui est général au point d'appartenir à tout langage et tel qu'on ne saurait concevoir un langage qui soit privé.* » in Frege Gottlob, *Les fondements de l'arithmétique (1884)*, trad. fr. Claude Imbert, Paris, seuil, 1971, p. 20.

These different definitions and explanations of logic lead us to the revolution of logic. Does the revolution of logic also differentiate as its definitions?

> "*The history of western logic can be divided into five period: 1. the ancient period (the 4th BC to the 6th century A.D); 2. the high middle Age (the 7th to the 11th century); 3. the Scholastic period (the 11th to the 15th century); 4. the older period of modern classical logic (16th to the 19th century); 5. the mathematical logic period(19th century still today).*"[12]

These five periods of logic (ancient, middle age, Scholastic, modern, and 19th century) constitute the various moment in which logic was been revolutionize or change of its direction.

- The ancient periods; the ancient period of logic starts from the ancient century Before Christ to the 7th century after Christ. One of this ancient area in which logic has been developed is the Greek. It is the Greek who in all the western region started developing the question related to the rules of thought. And this started before Aristotle but for a better and well understanding, we will start with Aristotle for whom most philosophers considered him as the father of logic:

> *"the problematic of formal logic by and large began with Aristotle. He was undoubtedly the most fertile logician there has never been, in the sense that a great many logical problems were raised for the first time in his work"*[13]. This work consist of "*furnishing the rules on which all reasoning is constructed; logic, without entering into all the phenomena of the mind, provides certain forms which an argument, to be legitimate, must exhibit, certain tests by which fallacy may be detected, and certain barriers against ambiguity in the use of language*."[14]

These certain tests by which fallacy may be detected are the fundamentals principles which lied at the basis of human reasoning. These include: the principle of identity, the principle of non-contradiction and the principle of excluded middle terms. Apart from this, Aristotle developed many others.

- We have the high middle age: this is a period of logic going from the 7th century to the 11th century. This period is considered to be uncreative due to the rise and domination of Christianity and the lots of Aristotelian books. This rise began in the 4th century with the

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[12]Joseph Maria Bochenski, *A History of formal logic*, trans. en. Ivo Thomas, Notre Dame, University of Notre Dame de Presse, 1961, Introduction, 3 Evolution of formal logic.
[13] *Id.*
[14] Aristotle, *Organon or logical treatises*, trans. en. Octavius Freire Owen, London, George bell & son, 1889, Introduction.

conversion of Constantine the roman emperor of that time. This conversion made the emperor to christianise all his territories and take over (burn) most of the philosophical books in the various libraries:

> "*the chief reason for the almost wholesale subordination of philosophy to theology in the middle age is that after the abolition of the school of Athens, the Platonic Academy by the emperor Justinian and his proscription of the teaching of pagan philosophy, intellectual activity fell under the authority of the church.*[15]

This means that, the church was at the center and control of any epistemological activity that had to be taken place. Any invention, discovery or research must be authorized by the chairman of the church who gives his authorization to carry out the activity. Everything had to passed through the church before this could be taught to the citizens. Some of the proponents of this period were: Boethius, Anselm and Averroes.

- The Scholastic period: The Scholastic which start from the 11th century which began by linking themselves to the antiquity and thus far simply took over and develop what was old. But from the end of the 12th century they started to construct some things entirely new. This logic which is properly theirs is almost all formulated metalogically. It is based on and accompany by an accurate and well developed semantic. For formulae consist, of words from ordinary language, with no or little variables, but their results narrowing of the semantic function as in the antiquity. Scholastic logic is accordingly thorough-going attempt to grasp formal law express in natural language with plentifully differentiated synthetical rules and semantic function.[16] Looking upon some of the precursor of this period we have; Peter Abelard, rivers kiwardby, William of Ockham, Ralph Strode, Paul of Venice and Peter Tarleret.

- The older period of modern classical logic also known as the transitional period is a period under the growth of science and mathematics. Here, logic is given a new direction that of a method of searching truth in science. As said the author of the *Introduction to logic*, that this period was an uncreative period, we can say that this is not totally right because even if logic was not more interested in the domain for establishing rules to guide our thoughts, this was instead use as a method to guide truth in science. So, "*with the renaissance and the development of the physical sciences and mathematics, with the attempts to constitute logic into*

[15] Anthony Clifford Grayling, *The history of philosophy*, New York, Penguin Publishing Group, 2019, the medieval and renaissance philosophy, p. 36.
[16] Joseph María Bochenski, *Op.cit*, p. 12.

a method ensuring the efficiency of scientific work, logic certainly do not go dormant"[17] as though Bochenski. The only thing that happens with logic was the change in its definition from that of the law of thought to that of the law of science. That is why, Descartes best known work, *the Discourse on the Method for Properly conducting reason and searching for truth in the science*"[18] is one of the examples. Throughout this book, the author formulated four rules that will help him to seek for certainty in the field of epistemology: "*likewise, in place of the large number of precepts of which logic is composed, I believed that the following four rules would be sufficient for me."*[19] This is because *"logic, its syllogisms and the greater part of its other lessons served more to explain to someone else the thing one knows, or even, like art of Lully, to speak without judgment concerning matter about which one is ignorant, than to learn them.*" [20]. We have to note that apart from Descartes considered as the father of this period, we have also Bacon, Leibniz one of the precursors of symbolic logic and Kant.

The second type of logic which is the modern logic or symbolic logic is made up of one period that of the mathematical logic period. This is the period whereby "*mathematical thought and logical thought thus arise in the same side and they belong to the world of pure understanding*"[21]. Symbolic logic was founded around the middle of the last century (XIX) and carried on into the present more by mathematicians than philosophers. The reason for this lies in the historical fact that during the past century, mathematicians became increasingly more conscious of the need to reexamine and reconstruct the foundation of the whole edifice of mathematics. Finding the traditional logic, a totally inadequate instrument for this purpose, the mathematicians set about to develop a system of logic that was at once more appropriate, more accurate and more comprehensive.[22]

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[17] This is our translation of : *« Avec la Renaissance et le développement des sciences physique et des mathématiques, avec les tentatives pour constituer la logique en une méthode assurant l'efficacité du travail scientifique, la logique ne s'est certainement pas mise en sommeil* », in Jaromir Danek, *Les projets de Leibniz et de Bolzano : deux sources de la logique contemporaine*, Québec, Les Presses Universitaire de Laval, 1975, p. 93.
[18] Anthony Appiah Kwame, *Thinking it through*, New York, Oxford university Press, 2003, p. 5.
[19] René Descartes, *Discourse on method (1637),* trans. en. Donald A. Cress, Cambridge, Hackett Publishing Company, 1998, p. 10.
[20] *Ibid.*, p. 11.
[21] This is our translation of « pensée mathématique et pensée logique (lesquelles) se rangent ainsi de même cote appartiennent au monde de l'entendement pur », in Ernst Cassirer, La philosophie des formes symboliques (1929), trad. et ind. Claude Fronty, Paris, Minuit, 1972, p. 40.
[22] Rudolph Carnap, *Introduction to Symbolic and its Applications*, trans. en. William H. Meyer & John Wilkinson, New York, Dover Publications, 1958, p. 3.

This inadequate of the traditional logic hold in "*the fact that logic has hitherto always followed ordinary language and grammar too closely"*[23]*. In addition, these natural languages have a superabundance's of words known to be blurred […] and this has nothing to do with science where every word need to be well determined."*[24] *"the natural languages admit many equivocal terms: the ambiguity, so dear to poetry and the spiritual man, abymes science and must be hated by scientists."*[25].

We should mention here that, the fact that symbolic logic was founded in the last century does not inferred that the idea was been develop in this period. The idea of such logic was already been established in the old modern classical period in the work of Leibniz as affirm by Frege: "*among the various sorties Leibniz made upon his goal, the beginning of symbolic logic come closest to what seem to be indicated by the phrase 'calculus rationatur*."[26]

> Therefore, *G.W. Leibniz is generally rank as the original mathematical logician, but if he cannot count as the founder of mathematical logic it is because his logical work were for the most part published long after his death (the essential by L. Couturate in 1901)*[27] and *what we do have are essentially only the magnificent fragment on the basis of which we can reconstruct his conception of this type of logic*[28].

After this, it is later that the work of Boole was been done to continue the development of mathematical logic in his first pioneer work, *The Mathematical analysis of logic* published in 1847. But logic came to it revolution mostly by the German mathematician and logician of the XIX century Gottlob Frege:

> "*Unquestionably the greatest genius of modern logic of the 19th century was, however, the German mathematician Gottlob Frege (1848-1925). More than anyone else he contributed to the interpretation of basic mathematical concepts in terms of the fundamental concepts of logic which operate with exact determinations right from the start. The first one to do so, he raised the logical calculus to a level at which it turns into the "interlude" of which Leibniz had spoken. "*[29].

> "*It is with his work on the Begriffsschrift which is not a philosophical but logical treaty that constitute perhaps the greatest single contribution in logic ever made and it was, in any event, the mostly*

[23] Gottlob Frege, *L'Idéographie, un langage formulaire de la pensée pure construit d'âpres celui de l'Arithmétique* (1879), trad. fr. Corinne Besson, Sorbonne V, Librairie Philosophique Jean Vrin, 2017, Préface.
[24] *Ibid.*, p. 124.
[25] *Id.*
[26] Gottlob Frege, *Posthumous writing*, United States, Basil Blackwell, 1979, p. 10.
[27] Joseph. María Bochenski, *Op.cit.*, p. 264.
[28] Heinrich Scholz, *Concise history of logic*, trans. en. F. Leidecker, New York, Philosophical library, 1961, p. 57.
[29] *Ibid.*, pp. 58-59.

> *important advance logic. In fact, its publication in 1879 can be considered a turning point in the history of logic, the birth of contemporary logic, sometimes called symbolic or Mathematical logic (...) But thought the Begriffsschrift is a logical work, it preface and Introduction contain interesting philosophical discussion, by the young Frege, an acquaintance with them is indispensable for a thorough understanding of the view of the mature Frege. "*[30]

Even if most of his work were not accepted by his contemporaries, this hasn't hindered Frege from attending his main focus and achievement that of creating a new logic called mathematical logic. That is why his "*intention was not to represent an abstract logic in formula, but to express a content through writing signs in a more précis and clear way than it is possible to do through words.*"[31] And he thinks that:

> "*the invention of this ideography has, it seems to him, advance logic. He hopes that logician if they do not allow themselves to be frightened by initial impression of strangeness, will not withhold their assent from the innovation that, by a necessity inherent in the subject matter itself, he was driven to make*".[32]

If the Ideography seem to Frege advance logic and a solution to ordinary language, this is because its first purpose is to provide us with the most reliable test of the validity of chain of inferences and to point out every presupposition that tries to sneak in unnoticed, so that it origin can be investigated. He believed that in replacing the concept of subject and predicate by argument and function, respectively, this will stand the test of time. Furthermore, the demonstration of the connection between the meanings of the if, and, not or, there is, some, all, and so forth, deserves attention.[33] Frege originality is therefore based on two aspects: the first aspect is the invention of a completely original symbols and the second aspect is that by given an independent logic of the grammar received, it makes it possible to analyze the statements otherwise than the natural language suggest.[34]

In this case, by choosing our thematic: **Frege and the logical revolution: a reading of the Begriffsschrift,** we engage ourselves in bringing an enlightment into a fundamental problem that of the epistemological pertinence of the Fregean logical revolution and innovation. In other words, along all of our work, we will analyze its logicism: how this has led to a great

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[30] Rosado Haddock, *A Critical Introduction to the Philosophy of Gottlob Frege*, United Kingdom, Ashgate publishing company, 2006, p. 1.
[31] Gottlob Frege, *Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought*, New York, Lubrecht & Cramer, 1879, Preface.
[32] *Id.*
[33] Gottlob Frege, *Ideography*, preface.
[34] Gottlob Frege, *Les fondements de l'arithmétique (1884)*, trad. fr. Claude Imbert, Paris, seuil, 1971, pp. 19-20.

change in the domain of logic and the pertinences this may have. This led us to several questions which can be raise from our mind: firstly, what are the pre-Fregean foundation of logic? Of what relevance can be the ideography that Frege proposes to substitute the traditional logic? Is this logico-mathematical language sufficient enough to disambiguate the errors of ordinary language? In final, what are the various philosophical interests that this might have? These four questions will constitute the so close of our work.

To well conduct our reflection, we will adopt the analytico-critical method. Why this method? The analytico-critic is a method of two words: analysis and critics.

This therefore, means that our work will consist in the first part, to analyze the different conceptions of logic before the German mathematician and philosopher Frege which greatly influence him. Such an analysis will permit us to examine the traditional logic of Aristotle considered today as the father of logic in general and in particular formal logic. Aristotle create propositions, inferences and syllogism which conduct one reasoning. Although these laws or rules to guide our thought, his logic face some limits. In this same part will be analyze others pre-fregean philosophers like Descartes rejection of traditional in favor of the four rules, the Port royal logic, Kantian heritage of logic, the mathematical logic of Leibniz and Boole.

In the second part, after showing Frege critics on the logic of his predecessor especially that of the Aristotelian traditional logic, we will analyses Frege's proper logic known as the "*ideography*". The ideography can be considered here as Frege formula language which is the combination of logic and mathematic (arithmetic) usually called mathematical logic. As said Frege, his Ideography is advance logic. This advance logic which will be develop here through, the philosophical foundation of his "Conceptography"; analyses of symbols, laws of the ideography, role of symbols, the differentiation between his logic and that of Aristotle. Frege system of logic; truth tables, concept and though and square of opposition. The last will be Frege and his contemporaries or followers: the Vienna circle and Wittgenstein.

In the third and last part, we will in one hand to carry out a critical analysis on the Begriffsschrift in order to evaluate the pertinence of it through; the hinderance in founding a formal language due to mutation and multilingual. Frege as an anti-evolutionism. In the other hand, to bring out the epistemological pertinence that this can have in the socio-political aspect, linguistic and epistemological aspect. We will finish with the fregean revolution of logic and Africa today; how this can be a useful into the black people diving in dogmatism, mysticism and mythicism.

PART ONE: THE PRE-FREGEAN FOUNDATION OF LOGIC.

That from the earliest time logic has traveled this secure course can be seen from the fact that since the time of Aristotle, it has not had to go a single step backwards, unless we count the abolition of a few dispensable subtleties or the more distinct determination of its presentation, which improvements belong more to the elegance than to the security of that science. What is further remarkable about logic is that until now it has also been unable to take a single step forward, and therefore seems to all appearance to be finished and complete.[35]

[35] Emmanuel Kant, *Critque of pure reason* (1781), trans. en. Paul Guyer & Allen Wood, Cambridge, Cambridge university press, 1998, Preface to the second edition, p. 106.

The well understanding of our thematic will implies an important and necessary analysis "*in rupture of what*"[36] the thought of our author is investigating on. It is this rupture that we called the 'Pre-Fregean'[37] approach of logic. The Pre-Fregean approach of logic become therefore the conception of logic or the way of thinking before Frege and which greatly influenced his ideas on the conception of the "Conceptography". These Pre-Fregeans philosophers can be divided into three; we have the traditional logic which will be our chapter one. We have the modern Classical logic which will constitute our chapter two is developed by the modern philosophers and logician. The last will be the Mathematical logic which will take our chapter three. It is built by great philosophers, mathematicians and logicians of the contemporary period.

[36] Thomas Minkoulou, lecture on U.E Phi 421 *: logique et philosophie des sciences*, 2021-2022, not published.
[37] Gottlob Frege, *Begriffsschrift*, p. 3.

CHAPTER ONE: THE TRADITIONAL LOGIC

In this chapter, we engage ourselves to carry out and analyze the Aristotelian implementation of logic. This means that, our work in chapter one will consist in developing the logicism of the father of logic. The reasons for us starting with the founder of the Lyceum is not as if the great thinkers who came before Aristotle failed to reason logically or at time illogically. But because Aristotle was the first, however, to unearth the basic principles of logical inference, to codify them, and to move some way toward showing the fundamental relations which obtain between some of the main types of logical form[38]. To clearly elaborate his logic, we decide to sub-divide this chapter into three: first we will discourse on the foundation of formal logic. The second aspect will be to come out with the trainee system of logic and last is to present the limits of his logic.

1.1. THE FOUNDATION OF FORMAL LOGIC.

1.1.1. Aristotle and the formal logic.

Formal logic is the investigation on the structure of propositions and deductive reasoning by a method which abstract from the content of proposition which come under consideration and deals with their forms. This mean that, formal logic study the different forms in which a proposition can be possible. So, logic is formal, in so far as it is concerned merely with the form of thought, that is with our manner of thinking without taking into consideration the particular objects about which we are thinking. Such a conception of logic was founded by the great father of logic, Aristotle[39]. That is why father Copleston affirms: "*the Aristotelian Logic is often termed formal logic. Inasmuch as the Logic of Aristotle is an analysis of the form of though, this is an apt characterization"*[40] This point of view can be seen again with one of the historian of logic Bochenski for whom: "*when we examine, we find that there is one thinker who so distinctly marked out the basic problem of this residual domain (formal logic) that all*

[38] Christopher Shields, *Aristotle*, London and New York, Routledge, 2007, p. 118.
[39] Aristotle was born at Stagira, a seaport town of the colony of Chalcidice in Macedonia, in the year 384 B., fifteen years after the death of Socrates. His father, Nicomachus, was physician to the king of Macedonia, Amyntas III. He was a disciple of Plato for twenty years after which he created his own school "lyceum". He died at Chalcis sixty-two years later. One of his greatest achievements is his foundation of formal logic writing in his Organon.
[40] Frederick Copleston, *A History of western philosophy vol. I, Greece and Rome*, London, Bloomsbury, 1946, p. 277.

later western inquirers trace their descent from him: Aristotle."[41] Therefore, we can say that the logic of Aristotle or precisely, the logic established by him is a formal logic in so far as it deals exclusively with forms, more strictly with perfect form and only with such form. This can be illustrated with the example below:

All men are boys

Some hard workers are men

Therefore, some hard workers are boys.

The example above is a form of thinking where the conclusion is directly drawn from two premises already established. The Aristotelian forms are known as syllogism which will be develop on the paragraph below.

1.1.2. The Sophistic refutation

The establishment of formal logic by Aristotle was aim to remedy the problem of language. Before Aristotle, there was in Athens in the 5th BC a movement of thinkers called the Sophists. The sophists were itinerant intellectual who taught courses in various subject, speculated about the nature of language and employed rhetoric to achieve their purposes, generally this was to persuade or convince their interlocutors. Language issues are at the center of the debates. It is through these debates that the sophists claimed to be able to discourse of everything in such a way to persuade their hearers. It thus appears a purpose different from that of philosophy that was developed by their predecessor who mostly answer to the question "what is the origin of things". In a precise way, the sophistical approach was eristic. "*Are eristic those who deduce a contradiction from what appear to be authoritative opinions but are not, or which apparently deduce a contradiction*"[42] This means that the debate been done by the sophists has nothing to do with truth and ethics but this is just a means to dominate through the use of language the listeners.

Aristotle had for this reason, create formal logic which are the various forms in which a proposition can be said valid or invalid. In other word, the author put forward some rules which must be obey for an argument to be correct or incorrect. Let us listen to him: "*it is sufficient that Logic, without entering into all the phenomena of the mind, provides certain*

[41] Joseph Maria Bochenski, *A History of Formal logic*, Trans. En. Ivo Thomas, Notre Dame, University of Notre dame de Press, 1961, p. 2, Introduction.

[42] This is our translation of « *sont éristiques ceux qui déduisent une contradiction à partir de ce qui parait être des opinions qui font autorité mais n'en est pas, ou qui déduisent en apparence une contradiction* » in Aristote, Œuvres complètes, trad. fr. Pierre Pellegrin, Paris, Flammarion, 2014.

forms which an argument, to be legitimate, must exhibit, certain tests by which fallacy may be detected, and certain barriers against ambiguity in the use of language."[43] Some of these rules usually known as the law of thought which must be obey by any one elaborating an argument for it to be coherent, rigorous and valid are; the principle of identity (a thing is always the same in the course of the same argument), principle of non-contradiction (a thing can't be what it is and what it is not) and the principle of excluded middle.

1.2: ARISTOTELIAN SYSTEM OF LOGIC.

1.2.1. Classification of proposition.

Definition: A proposition is the declarative sentence which have the capacity of been true or false. Such a definition is true but this is incomplete. To complete this definition, we can say with Aristotle that "a preposition then is a sentence which affirms or denies something, and this is universal, or particular, or indefinite"[44]. This therefore, mean that a proposition must be a sentence which carry the capacity of asserting something as been true or false and this assertion is done in a group of class which is universal or particular. We distinguish four types of propositions:

***** The first type of preposition is the **categorical proposition**. By the word categorical, we mean the straight or direct judgment on something. The categorical becomes the direct affirmation or negation without any condition, and qualification. This is why Aristotle could say "one first proposition is affirmation; afterwards negation, and all of the rest (preposition) are one by conjunction"[45]. By referring to one first preposition he means the categorical preposition. We have four types of categorical preposition.

Quantity	Quality	Sign
Universal	Affirmative	A
Universal	Negative	E
Particular	Affirmative	I
Particular	Negative	O

[43] Aristotle, *Organon*, Introduction.
[44] Aristotle, *Organon or logical treatises*, trans. en. Octavius Freire Owen, London, George bell & son, 1889, p.80, vol.3 The prior analytics book 1, Chap. 1.
[45] *Ibid*., p.52, vol.2, On interpretation, Chap.v of enunciation.

***** The second type of proposition is the **hypothetical proposition**. The hypothetical proposition is that which conjoint the truth of one prediction to the truth of another. This mean that the hypothetical propositions are those sentences that link two ideas of premises usually called the antecedent to the other called the consequence. This antecedent and consequence are bind together by supposition express in the form "if…then…" For example, 'if Paul fail then he will repeat. The hypothetical proposition has two forms: the first is, same subject and different predicates, example: if a student is obedient then he will succeed and have long life. The second is different subject and different predicate, example, If Cameroon won the world cup then all African will be happy.

***** The third type of proposition is the **disjunctive proposition**. Also called alternative proposition, a disjunctive proposition is a sentence that disjoint the truth of one or several proposition to the truth of the others. This serves as a separation between two or more propositions through the alternative "either …or…" for positive and "neither …nor…" for negative. Example, 'Either you are a boy or a girl'. Contrary to the hypothetical proposition with two forms, the disjunctive proposition has three forms: same subject and different predicates, different subjects and same predicate and different subjects and different predicates.

* The last is the **modal proposition**. A modal proposition is that which brings the writer ideas into the proposition by expressing his attitude toward what he/she is saying. This therefore, means that modal proposition freely live writer or speaker influencing the affirmation or negation of the preposition. It is what he or she put into the proposition that give a meaning to it. Modals proposition has four modes: "possibly, necessarily, impossible, contingence,"[46]. Example, it is not possible to be, it is necessary to be, it is possible to be, it is contingence to be.

1.2.2. Inferences

The square of oppositions of categorical propositions. The square of opposition of categorical proposition are the different ways in which the A, E, I, O are infer when oppose to each other. In other words, this is the theoretical relationship existing between two propositions with same terms, but which may differ in quality, in quantity or in both quantity and quality.

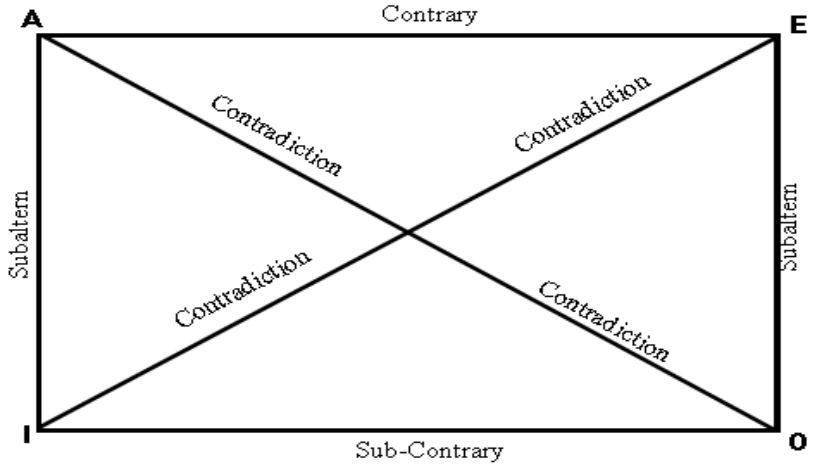
It should be reminded here that; the quantity of a proposition refers to the extent of the assertion made in the proposition. It tries to find out whether a proposition refers to "all" or

[46] *Ibid.*, p.72, vol. II.

"some" of the members of the class, or to an individual. We have three types of quantity of a proposition: the universal which include the whole of the class. This is represented by the word: "all", "every" "non", "No". For example, no girl is ugly; we have the particular proposition which include the part of a class or kind. This can be identified by words: "some" "except". For example, some boys are handsome; the last is the singular proposition which Aristotle attribute to the universal proposition. This proposition includes the individual or one thing only. For example, Douala is the head of the littoral region.

By quality of a proposition we refer to the fundamental character of a proposition. It determined whether a proposition is affirmative or negative. This affirmative and negative constitute the two types of quality of a proposition. Affirmative proposition asserts a proposition either totally or partially. Example, some boys are parent. The negation of a proposition is denying a proposition either totally or partially. Example, some learners are lazy.

The table below show us the diagrammatical representation of the immediate inference known as the square of opposition.



Subaltern: The subaltern is the logical relation that exist between two categorical propositions with the same subject and predicate terms, the same quality but different in quantity. Her the different pairs are: A-I, I-A, E-O and O-E. The rules of this opposition state that; from the truth of universal, we infer the truth of the particular and from the falsity of the particular we infer the truth of the universal.

Contrary: now, if any one universally enunciated of a universal, that something is or is not inherent, this enunciation will be contrary[47]. From this as affirm Aristotle, contrary become

[47] *Ibid.*, p. 55.

the logical relation that exist between two universal propositions negative and affirmative. Here, the propositions differ in quality but maintain their quantity. The pairs are: A-E and E-A. example: “Every man is white,” “no man is White”. The rule of this opposition is that, it is impossible that these should at the same time be true, one must necessarily be true and the other false.[48] The inverse is impossible.

Sub-contrary: when on the other hand he enunciates of universals, not universally, these are not contraries though the things signified may sometime be contrary.[49] This can be considered as sub-contrary. The pairs are, I-O and O-I. For example: “some men are white” “some men are not White” Here, the propositions have the same quantity but differ in quality. The rule here is that, one of the prepositions must be false and the other true.

Contradiction: We say affirmation is opposed to negation contradictorily, when the affirmation which signified the universal is opposed to that which is not universal[50]. Contradiction becomes the logical relation between universal and particular or particular and universal proposition. The pairs are, A-O, O-A, E-I and I-E. For example, “every man is white” “not every man is white” “no man is white” “some man are whites”. Here, the propositions change their quantity and quality. The rule here state that, of such contradiction then of universals, are universally made, one must necessarily be true or false. The table for the truth-value of the square of opposition can be represented on the table below.

Given that	Contrary	Subaltern	Contradiction	Sub-contrary
A is true A is False	E is False E is Doubtful	I is True I is Doubtful	O is False O True	Doubtful Doubtful
E is True E is False	A is False A is Doubtful	O is True O is Doubtful	I is False I is True	Doubtful Doubtful
I is True I is False	Doubtful Doubtful	A is Doubtful A is False	E is False E is True	O is Doubtful O is True
O is True O is False	Doubtful Doubtful	E is Doubtful E is False	A is False A is True	I is Doubtful I is True

[48] *Ibid.*, p. 56.
[49] *Ibid.*, p. 55.
[50] *Id.*

Apart from the immediate inference of the square of opposition we also have the Eduction: conversion, obversion, contraposition and inversion.

Eduction or Equivalence inferences

Conversion: This is also called the transposition where, by we interchange the position of the subject which become predicate and the predicate become subject. Concerning the application, the universal negative proposition is converted in its term "no pleasure is good" "neither will any good will be pleasure". But the affirmative proposition is converted into particular (accident), "all pleasure are goods" "a certain good should be pleasure". For particular propositions, we must convert the affirmative proposition particularly, "a certain pleasure is good" "a certain good be pleasure". The negative proposition need not to be converted, since it does not follow.[51]

Obversion: this is a process where we change the quality of a proposition, living it quantity unchanged and we attribute a contradictory to the predicate. In this inference, the original meaning of the proposition does not change. Here, the given proposition is the **Obvertend** and the inferred proposition is called the **Obverse. For example, Obvertend:** No learners are dull. **Obverse**: All learners are non-dull (intelligence).

Contraposition: This is a process in which we infer an equivalence proposition from a given Proposition by interchanging and contradicting the subject and predict terms. This means that the original subject takes a contradictory and become the predicate while the original predicate is contradict and become the subject. The contraposition undergoes three process; the first is Obverse, the second is the Converse and the last is the Obverse. **NB.** The A and O when contrapose remain the same, the E proposition is contraposed into the I and there is no contraposition of the I. **For example**: **Given**: All men are honest. **Obverse**: No men are dishonest. **Converse:** No dishonest are men. **Obverse:** All dishonest are non-men.

Inversion: This is the process of inferring an equivalence proposition by contracting it subject and predict terms and changing it quantity. The inversion inference undergoes through four processes depending on the given proposition. We have the **OCOC** for the A and the **COCO** for the E propositions. By **OCOC** we refer to **O**bversion, **C**onversion, **O**bversion and

[51] *Ibid.*, vol.3, the prior analytics, chap. II on the conversion of proposition, p. 83.

Conversion. By **COCO** refers to **C**onversion, **O**bversion, **C**onversion and **O**bversion. There is no contraposition of the I and O propositions. **For example**: **Given**: No Angels are mortals.

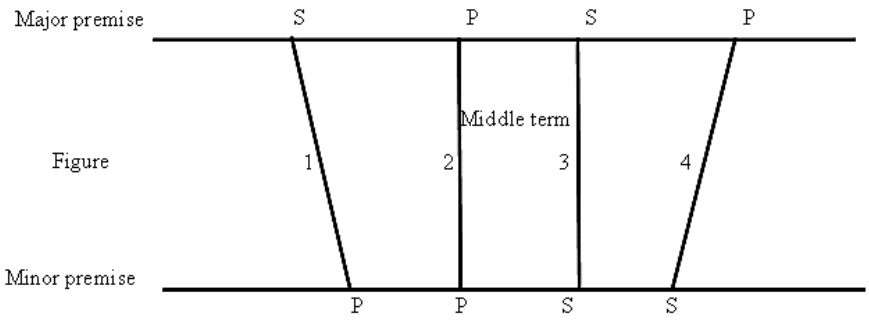
Converse: No mortals are Angels. **Obverse**: All Mortals are non-Angel. **Converse**: Some non-Angels are mortals. **Obversion**: Some non-Angels are not non-mortals. The eduction or equivalence inference can be summarized in the table below.

	Converse	**Obverse**	**Contraposition**	**Inversion**
A	Some P are S	No S are non-P	All non-P are non-S	Some non-S are non-P
E	No P are S	All S are non-P	Some non-P are not non-S	Some non-S are not non-P
I	Some P are s	Some S are not non-P	Impossible	Impossible
O	Impossible	Some S are non-P	Some non-P are not non-S	Impossible

1.2.3. Figures, laws, and modes of Aristotelian syllogism.

Definition of syllogism: A syllogism is a sentence in which certain premises being laid down, something else different from the premises necessarily results, in consequence of their existence[52]. By consequence of their existence, we mean that the conclusion of the premises is deduce from this premises, there is no need of any external term in order to the existence of the consequence. We have two types of syllogism: the perfect syllogism is that which needs nothing other than what has been stated to make plain what necessary follows.[53] The imperfect syllogism in which the conclusion is external to the premises already elaborated. Concerning the figures of a syllogism, we distinguish four of them. The diagram below shows us the different figures of syllogism.

Figures of syllogism.



[52] *Ibid.*, p. 82, Vol. III.
[53] *Id.*

- In figure one, the middle term is that which is itself in another, while another is in it. This means that in the first figure; the middle term is the subject of the major premise and predicate of the minor. For, A is predicate of every B (B is A), and B of every C (C is B), A must necessarily predicate of every C (C is A)[54].

M ─ T **For example**: All Human beings are mortals.

S ─ M Some boy are human beings.

S ─ T Therefore, some boys are mortals.

The rule is that, the major premise must be universal and the minor premise must be affirmative for a syllogism of figure one to appear.

- When the same (middle term) is present with every individual (of the one) but with none, (if the other) or present to every or none of each, a figure of this kind I call the second figure. This means that in the second figure; the middle term is both predicate of the major premise and minor premise.[55]

T ─ M **For example:** All girls are liars.

S ─ M No boy is a liar.

S ─ T Therefore, no boy is a girl.

The rule of writing figure two is that, the major premise must be universal and one of the premises must be negative for syllogism of the second figure to take place.

- When with the same thing one is present with every, but the other with no individual, or both with every or with none, such I call the third figure; and the middle in it, I call that which we predicate both.[56] This means that in figure three, the middle term is both subject of the major premise and minor premise.

M ─ T **For example:** No lion is an herbivore.

M ─ S Some lions are carnivores.

S ─ T Therefore, some carnivores are not herbivores.

Its rule is that, the minor premise must be affirmative and the conclusion particular, as illustrated on the diagram above.

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[54] *Ibid.*, p. 86, vol. III.
[55] *Ibid.*, pp. 89-90.
[56] *Ibid.*, p. 94.

- In figure four the middle term is predicate of the major premise and subject of the minor premise. "*The figure four was introduced in logic by the doctor Galen*"[57]. Aristotle does not say anything about this figure since he just elaborated the first three.

T ─ M **Example:** Some girls are prostitutes.

M ─ S All prostitutes are unfaithful.

S ─ T Therefore, some unfaithful person are girls.

In figure four the rule is, once the major premise is affirmative, the minor premise is universal.

The special rules of syllogism. The medieval logicians formulated eight (08) rules of syllogism which became classical. The first four (04) rules concern that of terms and the later four (04) are concern with proposition.

The first rule of concerning the terms is that, categorical syllogism must contain three terms; major, minor and middle terms and each of the terms appear twice in the syllogism. Once there are already more than three terms or a term appears more than twice, the syllogism commits the fallacy of "*Quaternio Terminorium*". Example,

1) All **boys** are **liars**
All **priest** are **boys**
Therefore, all **priest** are **liars**

2) All **boys** are **liar**
All **priest** are **prayers** Invalid
Therefore, all **priest** are **boys**.

The second is that, the middle term should not appear in the conclusion. Meaning that, the conclusion should not be made of the middle, but only of the major term and the minor.

Example; No lion is an herbivore. The middle term here is herbivore.
All goats are herbivores.
Therefore, no lion is a goat.

The third rule concern the distribution of the middle term. The middle term must be distributed at least once in the premises. If this is not the case, the proposition commits the fallacy of undistributed middle term. Example,

All living things are mortal.
All men are mortal.

[57] Roger Verneaux, Introduction général et Logique, Paris, Beauchesne et ses Fils, 1964, p. 100.

Therefore, all men are living things.

The table below show the distribution of terms on the four propositions.

Proposition	subject	predicate
A	D	U
E	D	D
I	U	U
O	U	D

D is Distribution and U is Undistributed. In the A proposition, the subject term is distributed and the predicate is undistributed. In the E proposition, the subject term distributed and the predicate distributed. For the I proposition, the subject term is undistributed and the predicate undistributed and in the O the subject is undistributed and predicate distributed.

The fourth rules of terms concern the distribution of terms in the conclusion. No term undistributed in the premises must be distributed in the conclusion. When this rule is violated, this commit the fallacy of either illicit process of the major or illicit process of the major. For example,

All women are mother or All learners are in universities.

No women are ugly person All learners are researchers.

Therefore, no ugly person is a mother Therefore, researchers are in universities

The first rule of proposition concern that of quantity. From two particular propositions we can't inferred any conclusion. Meaning that, no valid conclusion can be thrown from two particular propositions. The reason of this lies on the fact that, any combination of two particular propositions will commit either the fallacy of undistributed middle term or the fallacy of illicit process. For example,

Some boys are men.

Some boys are not courageous person.

Therefore, some courageous person are not men.

The second rule of proposition is that still related to this quantity. If one of the premises is particular, then the conclusion must be particular. This is to avoid the fallacy of illicit process of the minor.

The third rule of proposition is related to the quality of the proposition. This state that, from two negative proposition, no valid conclusion can be thrown. This is because, the negation denies class conclusion.

The last rule of the four and eight rule is that related to the quality. If one of the premises is negated, then the conclusion should be negated. This means that, once one of the premises is negated, automatically the conclusion thrown from such premises must be negated. This is to avoid that the fallacy of drawing and affirmation from a negation premise should appear.

Some students are hard workers.

Some hard workers are not wise.

Therefore, some wise persons are student.

The different moods and their names.

The moods are the way in which proposition are arrange in a syllogism. Each syllogism has three propositions and the representation of the letters of each proposition according to the order of premise: major, minor, and conclusion is the mood. In logic, we have twenty-four (24) moods which are valid, 19 of them are said to be "Strong" and 5 are said to be "Weak". A mood is said to be "weak" if the conclusion is too weak than it has to be in the premises. This means that, a weak mood is that which the conclusion is having a low extension than it premises. On the table will be represent the strong moods. Each figure has a number of strong moods and their names respectively.

Figures	Moods	Names
Figure one; four moods	AAA, EAE, AII and EIO.	B**A**B**A**R**A**, C**E**L**A**R**E**NT, D**A**R**II** and F**E**R**IO.**
Figure two; four moods	EAE, AEE, EIO and AOO.	C**E**S**A**R**E**, C**A**M**E**STR**E**S, F**E**ST**I**N**O** and B**A**R**O**K**O.**
Figure three; six moods	AAI, IAI, AII, EAO, OAO and EIO.	D**A**R**A**PT**I**, D**I**S**A**M**I**S, D**A**T**I**S**I**, F**E**L**A**PT**O**N, B**O**K**A**D**O** and F**E**R**IO**.
Figure four; five moods	AAI, AEE, IAI, EAO and EIO.	BR**A**M**A**NT**I**P, C**A**M**E**N**E**S, D**I**M**A**R**I**S, F**E**S**A**P**O**, FR**E**S**I**S**O**N.

1.3. THE ARISTOTELIAN LIMITS OF LOGIC.

1.3.1. Parallelism logic-language-metaphysics.

Parallelism is the character of what is parallel. The property of what is parallel is that they remain in at equal distances each other but we can easily establish a univocal reciprocal correspondence among the various points. This show that the traditional logic of Aristotle was establishing a correspondence between logic-language and metaphysic. According to him, we can easily move from grammar to logic and to metaphysics. It is in this light that Morgan reproached Aristotle to be too much the spokesperson for common language. In fact, for the tradition, the subject of logic corresponds to the grammatical substantive and represent a substance for a given reality. The copular correspond to the verb (to be) and represent the unique inference relation. The predicate logic corresponds to the grammatical attribute and represent the accidence that characterise the subject. So, the example "Paul is a boy" shows that boy is the intern modification and nature of Paul.[58] This is why Frege could write:

> "*these deviations from what is traditional find their justification in the fact logic has hitherto always followed ordinary language and grammar too closely. In Particular I believe that the replacement of the concept subject and predicate by function and argument, respectively, will stand the test of time.*"[59]

This means that the greatest error of Aristotle has been to attached logic to ordinary language and grammar which are ambiguity. So, it is important now to free our thought from all this misunderstanding and misleading expression that do not favour the enhancement of logic and science. This will be the aim of Frege Ideography.

1.3.2. The tautological aspect of traditional logic.

From its etymological word in Greek "*tautologia*" which is form from "*to autos*" (the same) and "*logos*" (discourse), this is the same discourse or telling the same thing. This means that, tautology is the analytical judgment in the sense that its content does not increase knowledge, contrary to synthetic judgment. Although tautology is generally perceived pejoratively, it sometime has merit of making explicit information already contain in a concept. [60] In its elaboration of judgment or proposition, the traditional logic was tautologous in that, the

[58] Philippe Nguemeta, Cours UE PHI 211 *: Introduction à la logique moderne*, Université Yaoundé I, Année académique 2022-2023, cours inédit.
[59] Gottlob Frege, *Idéographie*, pp. 8-9.
[60] Steeven Chapados, *Dictionnaire philosophique et historique de la logique*, Laval, Presse Universitaire de Laval, 2017, p. 469.

predicate which was use here was already contain in the subject or the conclusion of the deductive reasoning been enunciated was after all contain in the premises. This was saying the same thing in a different manner and because of this, there was no additional knowledge been added or advance:

> *The identity of the conceptions in analytic judgments may be either an explicit or an implicit one. In the former case the analytic prepositions are tautological. Tautological propositions are virtually empty, or void of consequence; for they are of no use whatever. Such is for instance, the tautological proposition, A man is a man. For if we can say nothing more of a man, than that he is a man, we know nothing more of him at all.*[61]

For example, the preposition All widows are unmarried person. Or the syllogism; All African people are black, All Cameroonian are African people, then All Cameroonian are black.

From the examples above, we can see in the proposition that, unmarried people which is the predicate is already contain in the subject widow since the nature of widows is been single. This is the same with the syllogism the conclusion: All Cameroonians are black people is already contained in both premises. This is the reason why Bertrand Russell affirms: *"in logic, every proposition is ultimately reducible to one which attributes a predicate to a subject. In any such proposition, the predicate is somehow contained in the subject. Thus, every true judgment of subject and predicate is analytic. That is, the predicate form part of the nation of the subject.*"[62] From this affirmation, it is clear that the traditional logic was a logic which was stable. This has a leg neither behind nor forward: "*besides, logic since the time of the stagyrite, has not gain much in point of master; nor can it do so from it very nature*"[63] of been tautologous.

1.3.3. The polysemy aspect of traditional logic.

From its original word "*polusemos*" meaning all that have several significations. Polysemy becomes the character of a word or concept which has more than one meaning or intention. When these meanings differ a lot between them, we say the concept is heterogeneous. The polysemic terms are more likely to create ambiguity when the use we do is not specified.[64] The ambiguity reside on the fact that the different meanings that we give to a word or concept create a total confusion into the psychologym of the individual. This can happen when someone hears/reads something without the same frame of references or contextual information as the

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[61] Emmanuel Kant, *Logic*, London, trans. en. John Richardson, Simpkin and marshall, 1819, p. 157.
[62] Bertrand Russell, *A critical exposition of the philosophy of Leibniz*, London and New York, Routledge, 1996, p. 11.
[63] Emmanuel Kant, *Logic*, Introduction, p. 23.
[64] Steeven Chapados, *Op.cit*., p. 302.

speaker/writer. Once a word or concept does not more have a precis meaning this make the language to wander in all direction and making the listener or reader to deduce his/her one understanding.

The Aristotelian logic is said to be polysemous because it was using the grammar of ordinary language which render some of the proposition to be confuse. For example, the proposition All women are light. This proposition is polysemous because the predicate "light" does not have precise reference. This term "light" can refer to the colour, the weight or shiny. This is why we can affirm with Michele Emanatian "*rarely do grammatical word have one and only one function.*"[65] This means that grammatical word always has several meanings. To Frege therefore, "*it necessary, or at least desirable, to develop a specialized language in which such defects would not hide. For example, the natural languages admit many equivocal terms.*"[66]

After a well elaboration of our chapter one, we can pose ourselves the question that follow: what can we retain from this? We should retain of this chapter one, firstly that, our preoccupation was that of the analyses of the traditional logic having as founder Aristotle. In this analysis, we have seen that the traditional logic is also called formal logic due to its nature of establishing rules and forms that guide our thought. This rules and forms can be seen on the various types of propositions, inferences, and syllogism with its laws, figures, rules, and moods. Secondly, it was to bring out the limits or weaknesses of this formal logic. Here, we gave some of it limits like it tautologous and polysemic nature which render it ambiguity. The limits of this logic dive us directly into a new conception of logic: the modern classical logic which will constitute our chapter two.

[65] Michele Emanatian, *Grammatical polysemy: The systematicity of multiple meaning in Grammar*, Doctorate thesis in philosophy, University of California at Berkeley, 1991, p. 1.
[66] Gottlob Frege, *L'Idéographie, un langage formulaire de la pensée pure construit d'âpres celui de l'Arithmétique,* p. 24.

CHAPTER TWO: THE MODERN CLASSICAL LOGIC AND IT PROBLEM.

In this chapter two which is carry on the modern classical logic, we will try to give the conception and the orientation of logic during the modern period. We have to know that this period goes from the sixteen centuries to the eighteen centuries. This was marked with a high increase in science especially mathematics and physics. Logic was not more regarded as the science of thought but the way to attain truth in science. To well apprehend this chapter, this will be divided into three sub-chapters: the first will be on the French philosopher Descartes and his rejection of formal logic, the second is the Port Royal logic and the last will be the Kantians logic.

2.1. DESCARTES AND THE REJECTION OF THE ARISTOTELIAN LOGIC.

2.1.1. The notion of method, evidence and certainty.

a. **Method :** the method is according to the Dictionnaire philosophique of André Comte un "*ensemble rationnel ordonné des règles en vue d'obtenir un certain résultat*[67]. This mean a method become any rational technique employed in the process of inquiring knowledge. A method is seen according to Descartes as that by "*which, it seems to me, I have the means to increase my knowledge by the degrees and raise it little to the highest point which the mediocrity of my mind and the short duration of my life will be able to allow it to attain*"[68]. The method is then the way or path which helps us to increase our knowledge when following it. The reason for the French philosopher to seek for a new method is not ex-nihilio but because when he was young, he had studied, among the parts of philosophy, a little logic which seemed, ought to contribute something in his plan. But in examine it, he notices that, in the case of logic, its syllogisms and a greater part of its other lesson served more to explain to someone else the things one knows, or even, like the art of Lully, to speak without judgment concerning matters about which one is ignorant, than to learn them. And although, in effect, logic might well contain many very true and very good precepts, he concluded that there are so many others

[67] André Comte-Sponville, *Dictionnaire Philosophique*, Quadrige, PUF, 2001, p. 586. The translation can be: ordered rational set of in order to obtain a certain result.
[68] Rene Descartes, *Discourse on Method* (1637), trans. En. Donald A. Cress, Cambridge, Hackett Publishing Company, 1998, p. 2.

mixed up with them that are either harmful or superfluous, that it is almost as difficult to separate the later from the former.[69] That is why he though it is necessary to search for some other method embracing the advantages of this logic yet free from its defects. He also reminds us that the purpose here is not to teach the method that everyone has to respect in order to conduct his reason well, but merely to show how he has tried to conduct his own.

b) **Evident** : « *une proposition est évidente si tout homme qui en a la signification présente à l'esprit, et qui se pose expressément la question de savoir si elle est vrai ou fausse, ne peut aucunement douter de sa vérité.* »[70] This means that, is evident what is impose to our though what can't be contested or denied, what then the truth appears immediately and can't be put into doubt. Descartes considered what is evident as that which present itself to our mind so clearly and so distinctly that we have no occasion to call it in doubt.

c) **Certainty:** A proposition is said to be certain when it is not suffering from any doubt that can contradict it truth. This is the reason why Descartes had a strong certainty on arithmetic because he considered them as been rigorous, coherent and precise. This can be seen when he affirms: "*I delighted most of all mathematics because of its certainty and the evidence of it reasoning.*"[71]

2.1.2. The four rules of the method.

> *Like wise, in place of the large number of precepts on which logic is composed, I believed the following four rules would be sufficient for me, provided I made a firm and constant resolution not even once to fail to observed them.*[72]

"*The first was never to accept anything as true that I did not plainly know to be such; that is to say, carefully to avoid hasty judgment and prejudice; and to include nothing more in my judgments than what presented to my mind so clearly and so distinctly that I had no occasion to call in doubt."*[73] From this affirmation, Descartes want us to carefully analyses each statement that come in to our mind so that we should not say something without distinguishing it through our faculty of reasoning. This is the rule that we called evident.

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[69] *Ibid.*, p. 10.
[70] André Lalande, *Vocabulaire technique et critique de la philosophie*, Quadrige, PUF, 1926, p. 310.
[71] *Ibid.*, p. 4.
[72] *Ibid.*, p. 11.
[73] *Id.*

"The second, to divide each of the difficulties I would examine into as many parts as possible and as was required in order better to resolve them"[74] This rule invite us not to solve problem in a whole but to always share each problem that we have into many parts so that this could be easier for us to well tackle the problem. This is the rule of analysis.

> "*The third, to conduct my thoughts in an orderly fashion, by commencing with those objects that are simplest and easier to know, in order to ascend little by little, as by degrees, to the knowledge of most composite things, and by supposing an order even among those things that do not naturally precede one another"*[75]

The third rule want us that after dividing the difficulties in to many parts, we solve them starting from those that are easier and simple and gradually increasing to solve those that are more complex. This is the rule of synthesis.

"The last, everywhere to make enumerations so complete and reviews so general that I was assured of having omitted nothing"[76]. After solving these difficulties, we must number them so that we should not forget none of them. This is the rule of enumeration.

2.1.3. The rule of universal mathematics.

Descartes in his book *Rules for the direction of the mind*, planned thirty-six rules but only twenty-one were been penned by him. These twenty-one rules can be divided onto two: the first twelve is on the principle of the scientific method which will be our main focus and the rest of the nine are specific to mathematics.

Concerning the first twelve rules on the principle of scientific method he says that: 1. Every study has an aim and according to him: "the aim of our studies must be the direction of our mind so that it may form solid and true judgments on whatever matters arise". This direction of our mind should not be base on knowing all objects but 2. We should attend only to those object of which our minds seem capable of having certain and indubitable cognition" and 3. "concerning these objects proposed for study, we ought to investigate what we can clearly and evidently intuit or deduce with certainty by first of all reading the works of the ancients to become aware of what scope is still left for invention in the various disciplines." To do these investigations, we 4. need a method 5. which consists entirely in the ordering and arrangement of the objects on which we must concentrate our mind's eye if we are to discover some truth.

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[74] *Id.*
[75] *Id.*
[76] *Id.*

By applying this method it will help us to distinguish the simplest things from those that are complicated and to set them in an orderly manner, 6. We should attend to what is most simple in each series of things in which we have had some truth from others and should observe how all the rest are more, or less, or equally removed from the simplest. 7. In order to make our knowledge complete, every single thing relating to our understanding must be survey through the principle of enumeration so that nothing should be omitted.

8. While examining the series of things we come across something which our intellect is unable to understand sufficiently well, it is important to stop at that point and try to understand it so that this will refrain from the superfluous task of examining the rest. 9. We must concentrate our mind's eye totally upon the most insignificant and easiest of matter, and dwell on them long enough to acquired the habit of intuiting the truth distinctly. Rule 10 have already been mention in rule 3. 11. If, after intuition a number of simple propositions, we deduce something else from them, it is useful to run through them in a continuous and completely uninterrupted train of thought, to reflect on their relations to one another, and form a distinct and, as far as possible, simultaneous conception of several of them. For in this way our knowledge become more certain, and our mental capacity is enormously increased. He concluded that 12. finally we must make use of all the aids which intellect, imagination, sense-perception, and memory afford in order, firstly, to intuit simple propositions distinctly; secondly, to combine correctly the matters under investigation with what we already know, so that they too may be known; and thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.[77]

2.2. PORT ROYAL LOGIC.

2.2.1. On the notion of ideas.

Ideas as defined Anauld and Nicole, *"are the simple view we have of the objects which are presented to our mind: as when, for instance, we think of the sun, earth, without forming any determinate judgement concerning them; and the form through which we consider them."*[78] From this definition, the port royal philosophers shows us that these ideas present in our mind are innate in us as defended by Descartes. For example, we have the idea of God because this

[77] The different rules mention above were been taken from Rene Descartes, *Rule for the direction of the mind*, trans. en. Laurence J. Lafleur, London, Liberal Arts Press, 1961.

[78] Anauld Antony & Peter Nicole, Logic *or the Art of thinking*, trans. En. Thomas Spencer, London, Simpkin, marshall and co, 1662, p. 25.

idea is in and is represent in our mind. There are three ways by which object are represented in our mind:

> *All that we conceive is represented to our mind, either as a thing, or as a manner of a thing or as a thing modified. I call a thing which is that we conceive as subsisting by itself, and as the subject of all which we conceive of it. This is what is known as the substance; as a manner of a thing which, being conceived in the thing, and cannot able to subsist without it. This is what is called the attribute or quality; as a thing modified which is when we consider the substance, as determine in a certain manner or mode.*[79]

Apart from representing things, our ideas can be universal, particular or singular. The single ideas can be divided into two; the ideas which only represent the single things are called singular or individual example, Socrates, Rome; and those represent many individuals are called universal, common or general example, man town, horse. Each word is generally into two: One which is called univocal is when words are connected with general ideas, so that the same word answer to many, both according to its sound, and ideas e.g. man, town the other is equivocal which is when the same word have been joined by men to signify several things or ideas for the same word. In this light, the word canon signified an engine of war, a degree of council, and an article of dress.

The five universal ideas: the first genus, also called genera this are ideas that extend to other ideas, although they themselves are universal.; the second is the species, which are those common ideas which are under one more common or general ideas.; the third is the differences, this is when the object of these ideas is an essential attribute, which distinguishes one species from another; the fourth, property, is when the object of a specie is an attribute, which belongs, indeed to the essence of the thing. But which is not the first we consider in that essence, but only depend on the first: as divisible, immortal. The property can be divided into four species: the first is that of "quod convenit omni, et soli, et semper" as every circle of the circle alone; the second is that of: "quod convenit omni, sed non soli" as divisibility is the property of extension; the third is "quod convenit soli, sed omni" as it belong to man to be taxi driver, though all men may not be so; the fourth is "quod convenit omni et soli, sed non soli" example is the change of the colour of the hair to grey when old for all men; the fifth universal idea is accident which is when we connect a confused and indeterminate idea of substance with a distinct idea of some mode, that idea is capable of representing anything in which the mode,

[79] Anauld Antony & Peter Nicole, *Op.cit.*, p. 35.

can exist: as the idea of prudent, all prudent men.[80] The universal ideas are also made up of two things which are: the comprehension and extension.

2.2.2. On the nature of judgment.

A judgment is that operation of the mind through which joining different ideas together, it affirms or denies the one of the other; as when, for instance, having the ideas of the Earth and Round, it affirms or denies of the earth that it is round.[81] This is what we called a proposition. According to them, proposition which is a sentence that can be affirmative or negative, and denoted by the verb is, are composed of three part; two terms which are the subject and predicate and the two terms are bound by the verb 'is' either affirmative or negative. This proposition is classified under quantity and quality, where the quantity determines the extension of the proposition either universal or particular and the quality determine the affirmation or negation of a proposition. The combination of the quantity and quality of a proposition give us four propositions with their vowel letters. See diagram 1 under Aristotle logic.

Apart from the Aristotelian compound (complex) propositions (conditional, disjunctive modal) seen above, the philosophers of port royal went further to add others: the **causal propositions** are those which contain two propositions connected by a causal particle, as 'or', 'because'. For Example: they are able, because they believe they are able. The truth of these propositions is that part be the cause of the other. The **relative proposition** which involve comparison and an example is where the treasure is, there the heart is also. Here the truth depends on the justness of the relation. The **discretives propositions** are those in which we make different judgments, denoting that difference by the particle but, nevertheless. In these sorts of propositions, the truth depends on the two parts, and separation that is made between them. The **exclusive propositions** are those which indicate that attribute agree with the subject, and that it agrees with that subject only, as virtue alone is true nobility; the exceptive propositions are those in which we affirm a thing of a whole subject, with the exception of certain inferiors of that subject, to which we show, by some exceptive particle, that this belongs.; comparative proposition in which we compare certain two judgments; the inceptives or desitives propositions are when we say a thing has commenced or ceased to be such.

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[80] *Ibid.*, p. 54.
[81] *Idid.*, p. 25.

2.2.3. On the nation of reasoning.

The authors defined reasoning as: "*that operation of the mind through which it form one judgment from many others; as when, for instance, having judge that true virtue ought to be referred to God, and that the virtue of the heathens was not referred to him, we conclude that the virtue of the heathens was not true virtue*."[82] This is what is mostly called in Aristotle logic "*syllogism*". This part which comprehend the rules of reasoning is regarded as the most important in logic, and almost the only which have been treated with care. In this sub-chapter little will be developed because most of the ideas been elaborated here have already been develop on Aristotle logic mostly concerning rules, figure and modes of a syllogism.

The study of logic helps men to well conduct their reasoning, that is having a good faculty of judging between correct and incorrect argument but the fact we study this discipline does not make us good logician because we can still be doing logic and reason badly. It not difficult in this way to analyse bad reasoning and this bad reasoning is what we call in logic sophism or paralogism. The argument been made from bad reasoning is known as fallacy. We have: 1) the fallacy of ignoration elenchi, this means been ignorant of that which ought to be proved against an adversary. 2) The fallacy of a begging of the question, which is clear altogether opposed to true reasoning. 3) Fallacy of non causa pro-causa committed when one mistake what is not the cause of an event to be the real cause. 4) The fallacy of incomplete enumeration committed when one makes imperfect enumeration by not sufficiently considering all the ways.

5) Fallacy of accident is when one tries to apply a general rule to a specific case that those not fall under it. 6) Fallacy of division which is the reasoning from the property of a whole to that of a part. 7) Fallacy a dicto secumdum quid ad dictum simpliciter. 8) Fallacy of ambiguity which are those committed when word or phrase do not express clear and precise meaning. They are of five kind, equivocation, amphibody, accent, composition and division. 9) Fallacy of hasty generalization which is the examination of many particular things, to rise to the knowledge of a general truth. This is called induction. Apart from the nine fallacies enumerated above, we have other kinds of fallacy such as: argumentum ad misericordia, acguement ad hominem, argumentum ad populum, argumentum ad vericundian, argument ad ignorantian, fallacy of sequitur, complex question, petition principii, fallacy of tu queque.

[82] *Ibid.*, p. 25.

2.2.4. On the nature of method.

The method is the operation of the mind, by which, having on the same subject (the human body, for instance) different ideas, judgment, and reasonings, it disposes them in the manner best fitted for obtaining a knowledge of the subject. This is the last part of logic develop by the Port royal philosophers. This may be called in general the art of disposing well a series of many thoughts, either for the discovering truth when we are ignorant of it, or for proving it to others when it is already known[83]. There are two methods as they propose:

> *thus there are two kinds of method, one for discovering truth, which is called analyses, or the method of resolution, and which may also be called the method of invention; and the other for explaining it to others when we have found it, which is called synthesis, or the method of composition.*[84]

The method of analysis is when we seek the cause of a thing through its effect, or the effect through the causes, or the whole from its parts or the parts from the whole. An example of analysis is when we find the area of a triangle from it high and bases. They gave four distinctions between an analysis and synthesis: the first is that the method of synthesis always pass from that which is more known to that which is less while in the analysis there is no such rule; the second is that the rules in analysis are establish according to their need contrary to synthesis where the rules are first establish; the third is that the truth known in the particular examination is taken from the thing which we are suppose to know and not from things more general, as do in the method of synthesis; finally, these two differ only as the road by which we ascend from a valley to a mountain from that by which descend from mountain to valley. The method of synthesis which consist principally in commencing with the most general and simple things, in order to pass to those which are less general and more complex. In this method, we mostly used rules, definition and demonstration.

2.3. THE KANTIAN LOGIC HERITAGE.

2.3.1. Kant theory of knowledge.

Kant theory of knowledge is based on one of the four questions which all interest of his reason (the speculative as well as the practical) is united. These questions are, *what can we know? What ought we to do? What may we hope for? What is man? The first question is answered by metaphysic, the second by philosophy, the third by religion and the fourth by*

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[83] *Ibid.*, p. 303.
[84] *Id.*

anthropology [85] His theory of knowledge is summarized in the first question that of what can I know? We have to know that what Kant called metaphysic is today what is philosophy (epistemology). In trying to answer to this question, Kant wrote a book entitle Critic of pure reason. In this book, Kant affirms that:

> *our cognition arises from two fundamental sources in the mind, the first of which is the reception of representations (the receptivity of impression), the second the faculty for organizing an object by means of these representation (spontaneity of concepts), through the former an object is given to us, through the latter it is though in relation to that representation (as a mere determination of the mind). Intuition and concept therefore constitute the elements of all our cognition, so that neither concepts without intuition corresponding to them in some way nor intuition without concept can yield a cognition. Both are either pure or empirical. Empirical, if sensation (which presuppose the actual presence of the object) is contained therein; but pure if no sensation is mixed into the representation. One can call the latter the matter of sensible cognition. Thus, pure intuition contains merely the form under which something is intuited, and pure concept only the form of thinking of an object in general. Only pure intuitions or concepts alone are possible a prior, empirical ones only posteriori.*[86]

From this quotation, Kant reconcile the two epistemological approach of knowledge which were opposed each other. We have the rationalists and the empiricists. The rationalists which had as father Descartes hold that the only source of our knowledge come from our reason and that each of us is endow with this at born. This therefore means that, man can know all through the action of reflection. Contrary to the rationalists, the empiricists which has as proponents John Locke and David Hume hold that our knowledge is as a result of experience. This means that, at born our mind is a blank slate and it is only when our five senses got experience that something is writing into our mind. Kant criticizes these two approaches of knowledge by saying of rationalism that "*la croyance aux idées innées ou aux vérités éternelles est la plus grosse erreur des philosophies dogmatiques*"[87] while empiricism is sceptical. If reasoning alone cannot lead us to all truth because there are some things in which reason cannot attain and empiricism cannot also lead us to all truth because there are some things in which this can't be reach. This therefore means that knowledge is as the result of the unity of intuition with concepts of the understanding, as well as objects of experience should be structure by the understanding. Once more let us listen to Kant :

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[85] Emmanuel Kant, *Logic*, Introduction, p. 30.
[86] Emmanuel Kant, *Critque of pure reason* (1781), trans. En. Paul Guyer & Allen Wood, Cambridge, Cambridge university press 1998, p. 193.
[87] Emmanuel Kant, *Critique de la raison pure,* trad. Fr. Tremesaygues et B. Pacaud, Paris, PUF, 1944, préface.

> *C'est n'est qu'en s'unissant que l'entendement et la sensibilité peuvent déterminer en nous des objets. Si nous les séparons, nous avons des intuitions sans concepts, ou des concepts sans intuitions, et, dans les deux cas, des représentations que nous ne pouvons rapporter à aucun objet déterminé*.[88]

From this text, it resorts that both empiricism and rationalism are complementary and that neither of these properties is to be preferred to the other. Without sensibility, no object would be given to us, and without understanding none would be though. Therefore, as affirm Kant; "*though without content are empty, intuitions without concept are blind*."[89]

Kant distinguishes two types of knowledge: a posteriori knowledge and a priori knowledge. A priori knowledge is that which is independently of experience, because they make one able to say more about the objects that appear to the senses than mere experience would teach, or at least make one believe that one can say this, and make assertions contain true universality and strict necessity, the like of which merely empirical cognition can never afford. He considered such knowledge to be rational: "*only pure intuition or concept alone are possible a priori*."[90]

2.3.2. Kant theory of judgement.

A judgment to Kant "*is the representation of the unity of the consciousness of various representation, or the representation of their relation, provided that they make up a conception.*"[91] This means that judgment is therefore the mediate knowledge of an object, consequently the representation of the concept of that object. The father of criticism distinguishes two types of proposition: the **analytic proposition** and the **synthetic proposition**. "*Those propositions, whose certainty depends upon the identity of the conceptions (of the predicate with the notion of the subject), are analytical. Those, whose certainty is not founded in that identity, must be named synthetic*."[92] By defining an analytic proposition as identity of the concepts subject and predicate, this means that the analytical proposition does not add any knowledge. For example, 'All men are boys. From this example, it is seen that the nature of the concept boy is already contain in men. Therefore, the analytical proposition is tautological: "*tautological propositions are virtually empty, or void of consequence; for they are of no use*

[88] Emmanuel Kant, quoted by Tokam Félicien, in his dissertation : *les catégories de l'entendement : fondement de la connaissance chez Kant*, defended in the University of Yaoundé I, Academic year 1991-1992, p. 21.
[89] Emmanuel Kant, *Critque of pure reason* (1781), pp. 193-194.
[90] *Ibid.*, p.193.
[91] Emmanuel Kant, Logic, p. 141.
[92] *Ibid.*, p.156.

whatever."[93] The synthetic proposition are makes our knowledge to progress because this is not identical leading to tautology like the analytical proposition.

Apart from these two distinguished propositions, Kant added other propositions: the **expoundable proposition, theoretical and practical proposition** and **indemonstrable and demonstrable proposition**. At the end, the author of Logic came out with four categories instead of the ten been proposed by Aristotle.

CATEGORIES	NATURE	PROPOSITION	EXAMPLE
QUANTITY	Unity	Universal	Man is mortal
	Plurality	Particularity	Some men are mortal
	Totality	Singular	Plato is mortal.
QUALITY	Reality	Affirmative	The soul is mortal.
	Negation	Negative	The soul is not mortal.
	Limitation	Indefinite	The soul is mortal.
RELATION	Substance and accident	Categorical	This boy is black.
	Cause and Effect	Hypothetic	If a body hot itself it dilates.
	Reciprocity	Disjunctive	A substance is either solid or liquid or gaseous.
MODALITY	Possibility	Problematic	It may rain.
	Existence	Assertory	Its rain.
	Necessity	Apodictic	A substance is necessarily extended.

2.3.3. Subjectivism and Objectivism in Kant theory of knowledge.

a) Subjectivism

As defined André Lalande, Subjectivism is : « une *tendance philosophique qui consiste à ramener tout jugement de valeur ou de réalité a des états de conscience individuels* »[94].

[93] *Ibid.*, 157.
[94] André Lalande, Op. cit., p. 1039.

Meaning that subjectivism is subject which refers to individual or singular. This are truths which depend on the individual who is the measure of all things. Such a conception of reality is been rejected by Kant. One of the subjective truths rejected by Kant is the aesthetic judgment which to him is "*a merely subjective truth, which consists but in the agreement of the cognition with the subject and with the laws of the appearance of the sense, and by consequence is nothing more than universal appearance*."[95] Aesthetic can be defined as the branch of philosophy dealing with the beauty or the beautiful, especially in art, and with taste and standards of value in judging art. Kant main rejection of aesthetic judgment is the judgment of taste, that is judgment about the beautiful, above all about the beautiful in nature. According to the author of *Critique of judgment*, the judgment of taste is not a judgment of knowledge. We can hear him talking: "*hence a judgement of taste is not a cognitive judgment and so is not a logical judgement but aesthetic one, by which we mean a judgment whose determining basis con not be other than subjective*."[96] Lalande from this quotation, it is clear that aesthetic truth depend on the individual and this cannot be considered as a universal truth which can be accepted by all. Since taste is not a cognitive knowledge, this cannot be a form of knowledge. The author argue that taste is not a cognitive knowledge in that:

> *if we wish to decide whether something is beautiful or not, we do not us understanding to refer the representation to the object so as to give rise to cognition; rather, we use imagination (perhaps in connection with understanding) to refer the representation to the subject and his feeling of pleasure or displeasure*." [97] And "*the subjective of a representation which cannot at all become element of cognition is the pleasure and displeasure connected with that presentation.*[98]

This therefore means that our truth of aesthetic does come from our faculty of judging between good and bad but this is the act of imagination which is based on pleasure. Hence a judgment of taste, which involve this pleasure is like any empirical judgment because it cannot proclaim objectives necessity or lay claim to a priori validity; but, like any other empirical judgment, a judgment of taste claim only to be valid for everyone, and it is always possible for such a judgment to be valid for everyone despite its intrinsic contingency. What is strange and different about a judgment of taste is only this; that what is to be connected with the presentation of the object is not an empirical concept but a feeling of pleasure (hence no concept at all), though, just as if it were a predicate connected with cognition of the object, this feeling is

[95] Emmanuel Kant, *Logic*, p. 51.
[96] Emmanuel Kant, *Critique of jugement (1790)*, trans. En. Werner S. Pluhar, Cambridge, Hackett Publishing Company, 1987, p. 44.
[97] *Id.*
[98] *Ibid.*, p. 29.

nevertheless to be require by everyone. In order word, the pleasure in a judgment of taste is indeed dependent on an empirical presentation and cannot be connected a priori with any concept; but we are aware that it rests merely on reflection which are only subjective condition.

b) The valorization of objectivism and rejection of Aristotle formal logic.

Objectivism is the doctrine maintaining that everything apprehended is independent of the apprehender. Meaning that knowledge does not depend on individual but is objective, the Kantian theory of objectivism is effectively in this perspective. By examine the a priori element that render the intelligibility of knowledge possible, Kant illustrate that objectivity is not only found on the relationship to things, but also the relation to the mind. It guarantees the constancy and uniformity of knowledge from one individual to another. This constitute what Kant called objectivism. Since a priori signifies not only what comes before any experience, but also what is absolutely independent of all experience, there is no doubt that it is an essential factor in our cognition process, in order to attain knowledge that is at once rational and objective. This is the reason why subjective apprehension should be distinguished from objective appearance as affirm Kant:

> *"I must therefore derive the subjective apprehension from the objective appearances, for otherwise the former would be entirely undetermined and no appearance (objective) would be distinguished from any other. Subjective apprehension alone proves nothing about the connection of the manifold in an object, because it is entirely arbitrary. This connection must therefore consist in the order of the manifold of appearance in accordance with which the apprehension of one thing follows that of the other in accordance with the rule. Only thereby can I be justified in saying of the appearance itself, and not merely of my apprehension, that a sequence is to be encountered in it, which is to say as much as that I cannot arrange the apprehension otherwise than in exactly this sequence.*"[99]

The passage above just shows us that subjectivity should be separated from that of objectivity because all sequence of perception is determined solely in apprehension, that is, merely subjectivity, and would not thereby be objectively. Kant objectivism therefore, consist in the affirmation and demonstration of the legitimacy of our representation. It does not say that the mind can directly know the noumena. In anyway, there is a connection between the notion of objectivity and Kant objectivism. It is in some way related to absolute and nominal aspect of objectivity, but not perfectly with experimental objectivity. Since objectivism also constitutes Kant's criticist method, one does not fail to notice the intellectual astuteness of Kant, his

[99] Emmanuel Kant, *Critque of pure reason* (1781), p. 307.

negation of narrow-mindedness, and his distaste of partisanship. As such, he did not explain only the possibility of morality based on reason. It substantiates the neutrality and rigor, as well as the freedom that characterizes all true philosophical enterprise.[100]

Kant rejection of traditional logic can be seen in his book entitle the *Critique of pure reason* where he clearly state that: "*logic since the time of Aristotle has not had to go a single step backward nor has also be able to take a single step forward and therefore seems to be finished and complete.*"[101] But through his new conception of logic, the philosopher of the transcendence will demonstrate that the formal logic was facing some limits which make it not to move backward nor forward. These limits will constitute the differences between the Aristotelian logic and Kantian logic. To well examine this differentiation, we will focus our selves in the categories of Kant already established on page 34 above. The author began by rejecting all the ten categories put on by Aristotle to considered just four; quantity, quality, relation and modality: "*Following Aristotle we will call these concepts categories, for our aim is basically identical with his although very distant from it in execution.*"[102]

This distant in execution is based on the fact that; firstly, the logicians rightly say that in the used of judgments in syllogism singular judgments can be treated like universal one. This mean that singular proposition should be range under universal propositions. But the transcendental philosopher think that: "*on the contrary we must distinguish a singular judgment with a generally valid one, merely as a cognition, with respect to the quantity it has in comparison with other cognition, then it is surely different from generally valid judgments and deserves a special place in a complete table."*[103] Secondly, in transcendental logic infinite judgments must also be distinguished from affirmative ones, even though in general logic they are rightly included with the latter and do not constitute a special member of the classification.[104] Thirdly, the disjunctive, the hypothetical and categorical which is considered in the traditional logic as the three types of propositions, is considered by Kant to the three form of relation. Lastly, the modality of a judgment is a quite special function of all the rest of the categories in that it is concern only with the value of the copula in relation to thinking in general. The problematic judgments are those in which one regard the assertion or denial as merely

[100] Ngewoh Christopher, in his memoire, *Kantian objectivity in the Critique of pure reason*, defended on the academic year 1982-1983, p. 106.
[101] Emmanuel Kant, *Critque of pure reason* (1781), 106.
[102] *Ibid*., p. 212.
[103] *Ibid.*, p. 207.
[104] *Id.*

possible. The assertoric judgements are those in which it is actual and the apodictic judgments are those in which it is necessary.[105]

Kant in his critique on the deduction of pure concepts of the understanding, defined three original sources for all possible experiences: "*there are, however, three original sources(capacities or faculties of the soul), which contain the conditions of the possibility of all experience, and cannot themselves be derived from any other faculty of the mind namely,* ***sense, imagination,*** *and* ***apperception.***"[106] Among these faculties, imagination, regarding its function, stands between the two (sense and apperception) and has a central role in cognition. Kant treat the imagination as a distinct faculty of the mind which work as a mediator between the sensibility and understanding. This means that the imagination appears as central to both a priori and empirical knowledge. As this can be seen in this affirmation:

> *we therefore have a pure imagination, as a fundamental faculty of the human soul, that ground all cognition a priori. By its means we bring into combination the manifold of intuition on the one side and the condition of the necessary unity of apperception on the other. Both extremes, namely sensibility and understanding, must necessarily be connected by means of this transcendental function of the imagination since otherwise the former would to be sure yield appearance but no object of an empirical cognition, hence there would be no experience.*[107]

Arriving at the end of this chapter two, we can retain from it that, our main preoccupation here has been to analyze the modern classical period following Descartes, Antony & Nicole and Kant. During our analysis, we showed that during this period most logic was given a new orientation through the research of truth in science as developed Descartes who criticize the traditional logic. It is in this same light that the philosopher of light also rejected the traditional logic by given out the four categories on which all its critics are founded. The next chapter will be focus on the formalization of proposition into mathematical logic.

[105] *Ibid.*, p. 209.
[106] *Ibid.*, p. 225.
[107] *Ibid*., p. 241.

CHAPTER THREE: LEIBNIZ, BOOLE AND MATHEMATICAL LOGIC

Mathematical logic also known as symbolic logic or modern logic is the discourse on mathematical symbols introduce in logic. In this logic, we discover how notations of arithmetic and algebra are been employed and used in logical propositions. This will then be our focus in this chapter three and to well see how mathematical signs and symbols are employed in logic, we decided to subdivide this part in four sub-chapters. The first will be to analysis the logic of Leibniz considered to be one of the precursors of this logic; the second is the three laws of Boolean algebra calculus; the third will continuous with the important of Boolean simplification; the last is the algebraic simplification of logic. Before starting, we have to note that, it is based on the work of these two authors especially Leibniz that most of the ideography of Frege will be built on. As he affirms "*it is possible to view the signs of arithmetic, geometry, and chemistry as realization, for specific fields, of Leibniz's idea. The ideography proposed here add a new one these fields.*"[108]. So, in most of this chapter, there will be a relation that will make us each time to illustrate our author Frege and the two main authors chosen here.

3.1. THE FOUNDATION OF MATHEMATICAL LOGIC.

3.1.1. The correction of traditional logic.

The traditional logic presented by Aristotle and the Scholastic were admitting four figures of a syllogism and in each figure had a number of modes: figure one four modes, figure two four modes, figure three six modes, and figure four five modes. Leibniz sees this logic of syllogism imperfect; he wants therefore to correct and complete it. Let us listen to the correction he made:

> *the logic of syllogism is truly demonstrative, just like arithmetic or geometry. I demonstrated in my youth not only that there are really four figures which is easy, but also that each figure has six useful moods, and can not have neither more nor less: instead that ordinarily we only give four to the first and second, and five in the third* [109]

[108] Gottlob Frege, *Ideography*, p. 7.

[109] This is our translation of « *la logique du syllogisme est véritablement démonstrative, tout comme l'Arithmétique ou la Géométrie. J'y démontré dans ma jeunesse, non seulement qu'il y véritablement quatre figures, ce qui est aisé, mais aussi que chaque figure a six modes utiles, et n'en saurait avoir ni plus ni moins : au lieu qu'ordinairement on n'en donne que quatre à la première et la second, et cinq à la quatrième* », in Louis Couturat, *la logique de Leibniz*, Paris, Felix Alcan, 1901, p. 2.

From the text above, it appears that the authors accept the four figures of the syllogism but was not for the different modes that were been given, that is why he proposes a new combination of moods which give us six modes per figure and the total give us twenty-four modes. These twenty-four moods can be deduced from the principle of regression or absurd reduction. Firstly, we demonstrate the four moods of figure one infers from a unique principle which Leibniz the "basic of syllogism" which can be name as *dictum de omni and dictum de nullo.* The dictum de omni is the affirmative moods: BABARA and DARII and the dictum de nullo is the negative moods: CELARENT and FERIO. The rest of the two moods of figure one is obtain by sub alternation: BABARI and CELARO.

Once this done, the six moods on figure one generates through regression the moods of figure two and three. Figure two is generated from that: we maintain the major premises of each moods(fig.1), we do the negation of the conclusion of each mood (fig.1) to obtain the minor premises and we negate the minor premises of each moods to have the conclusion. Figure three is infer in that: we do the negation of the conclusion of each mood (fig.1) to have the major premise, we maintain the minor premise and we negate the major premises to have the conclusion. The applicability of these rules can be seen on the different moods below.

AAA (fig.1) generate by regression the AOO (fig.2) and OAO (fig.3)

EAE (fig.1) generate by regression the EIO (fig.2) and IAI (fig.3)

EAO (fig.1) generate by regression the EAO (fig.2) and AAI (fig.3)

The remaining figure which is figure four is obtained not by regression but by conversion. Conversion is the interchangement of the subject and predicate. This mean that the subject becomes the predicate and the predicate become the subject. The table below sum up the twenty-four moods of Leibniz syllogism.

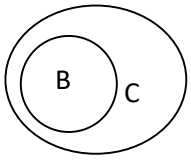
Figure	**Moods**	**Names**
One	EAE, EIO, AAA, AII, AAI, and EAO.	C**E**L**E**R**E**NT, F**E**R**IO**, B**A**B**A**R**A**, D**A**R**II**, B**A**B**A**R**I**, and C**E**L**A**R**O**
Two	EAE, AEE, AOO, AEO, EAO and EIO.	C**E**S**A**R**E**, C**A**M**E**STR**E**S, C**E**S**A**R**O**, C**A**M**E**STR**O**S, F**E**ST**I**N**O** and B**A**R**O**C**O**.
Three	OAO, IAI, AII, EIO, EAO and AAI	B**O**C**A**D**O**, D**I**S**A**M**I**S, D**A**T**I**S**I**, F**E**R**I**S**O**N, F**E**L**A**PT**O**N and D**A**R**A**P**I**.
Four	EAO, AEO, IAI, EIO, AAI and AEE.	F**E**S**A**P**O**, C**A**M**E**N**O**S, D**I**M**A**R**I**S, FR**E**S**I**S**O**N, B**A**M**A**T**I**P and C**A**M**E**N**E**S

Still in the correction of traditional logic, Leibniz in his paper "de Formae Comprobation per linearum ductus" probably written after 1686, elaborated two methods for representing the content of categorical propositions. This is why Couturat affirm on this purpose :

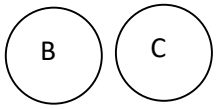
> *Leibniz cherchait a représenté les raisonnements, en particulier les syllogismes, les figures géométriques ; et il attachait une grande importance à ce schématismes… Il n'a pas seulement inventé avant Euler les schèmes circulaires de tous les modes du syllogisme ; il a aussi inventé un system de schèmes linéaires encore plus ingénieux et plus parfait.* [110].

The circular representation of each proposition is as follow:

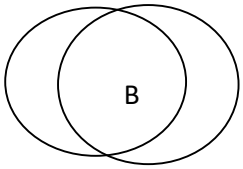
Universal Affirmative (U.A): All B are C



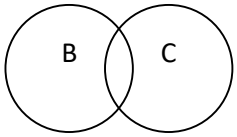
Universal Negative (U.N): No B are C



Particular Affirmative (P.A). Some B are C



Particular Negative (P.N). Some B are not C



3.1.2. The foundation of mathematics.

Although Leibniz approved and adopted the traditional logic by correcting and completing it, it will be fault to think that his logic is just the development and perfection of the Aristotelian logic. Indeed, his logic is more than what we can think of and this has been seen by Philalethe in his complement to Leibniz : *« vous paraissez faire l'apologie the la logique vulgaire, mais je vois bien que ce que vous apportez appartient à une logique plus sublime, à qui la vulgaire n'est que ce que les rudiments abécédaires sont l'érudition.* »[111] It is this sublime logic that Leibniz founded his mathematic. The Leibniz mathematic was based on what is called the **combination** which to him is the art of inventing. This consist in the multiplication and

[110] *Ibid.*, p. 25.
[111] Leibniz Gottfried, *Nouveau essaie sur l'entendement humain*, Paris, Ernest Flammarion, 1704, Liv. IV, Chap. XVII, $ 7, p. 434.

addition of concepts to find the possible outcome of a subject or predicate in a given term. How does it function?

First of all, we analyze the simplest concept and this will give us the first class. After, we arrange the concepts of the first class by combining two by two which will give us the second class. The third class is obtained by the combination of the concepts three by three and the process continuous. For example, the numbers 210 possible output are: in the first class the number: 2, 3, 5, 7 which is the simplest number by which 210 can be divisible; to obtain the second order we combine them as follow: 2.3, 2.5, 2.7, 3.5, 3.7, 5.7 ; the third order is combine as 2.3.5, 2.3.7, 2.5.7, 3.5.7. It should be note as Leibniz wrote: "*there are simple ideas, of which no definition can be given; there are also axioms and postulate, in a word, primary principle, concept, which cannot be proved, and indeed have no need of proof; and these are identical propositions, whose opposite involve an express contradiction*."[112]

3.1.3. The universal calculus as Leibniz founder of symbolic logic.

Before Frege, Leibniz brought his contribution in logic. This can be recognise once more by Philalethe in his affirmations : « *je commence à me former une tout autre idée de la logique que je n'en avais autrefois. Je la prenais pour un jeu d'écolier, et je vois maintenant qu'il y a comme une mathématique universelle de la manière que vous l'entendez.* »[113] This means that, Leibniz aim was to come out with a new logic which will be based on mathematical symbols and notations. To achieved this aim, he starts his universal calculus by given a rule *"la règle de composition des caractères est la suivante : un terme compose de plusieurs termes simple sera représenté par le produit des nombres premiers qui correspondent à ses termes simples. Par exemple, l'homme est un animal raisonnable : soit h=homme, a=animal, r=raisonnable.* »[114] From this, Leibniz shows us that each term must be represented by a given letter. The main elements of Leibniz's algebra concept which constitute it universal calculus (symbolic logic) may be summarize in the following diagram.

3.1.4. The aim of universal graphics symbols.

Leibniz aim of universal graphic symbols lies on the fact that learn men have long since though of some kind of language or universal characteristic by which all concept and things can be put into beautiful order. Meaning the universal graphic had to play the role of unity by

[112] Leibniz Gottfried, *The monadology* (1714), Trad. En. Robert Latta, 1898, paragraph 35, p. 5.
[113] Leibniz Gottfried, *Nouveau essai sur l'entendement humain*, Liv. IV, Chap. XVII, $ 8, p.436.
[114] Louis Couturat, *La logique de Leibniz*, Paris, Felix Alcan, 1901, p. 326.

forming a language in which people will be expressing their self in a mathematical way. This was to prevent any controversy between people as been done in ordinary language. Ordinary language creates confusion and ambiguity due to it polysemous nature which render language unprecise. Therefore, with the use of mathematical notation for each term and concept there will not more be any debate of contradiction. We can once more listen to him through the writings of Bertrand Russell "*if controversy were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their hand, to sit down with their slates and say to each other let us calculate.*"[115] The calculation been talk by Leibniz is not like that of pure mathematic but this refers to a manner of saying let talk like mathematicians using notation and symbols.

By comparing his work to that of his predecessors like Aristotle, he affirms: "*yet no one has attempted a language or characteristic which includes at once both the arts of discovery and judgment, that is, one whose signs or characters serves the same purpose that arithmetical signs serves to numbers, and algebraic signs for quantities taken abstractly.*"[116] Leibniz uses letters to denote concepts and special symbols to denote conjunction and quality of concepts. The analogy between concepts and numbers is clearer in his *Two studies in the logical calculus* when he gave an example showing that: since man is a rational animal, if the number of animal is a, for instance, 2, and the number rational is r, for instant, 3, the number man, or h, will be 2*3=6.[117]

The idea of a universal calculus been put on by Leibniz will be first of all been taken my Boole before been systematized by Frege. Boole will be the first mathematician who will transform logical proposition into mathematical notation with the help of algebra. Our next sub-chapters will be an analysis of his work mostly carry out in his book entitled "*Mathematical analysis of logic*"

[115] Bertrand Russel, *A Critical Exposition of the philosophy of Leibniz*, London and New York, Routledge, 1996, p. 170.
[116] Bruno Woltzenlogel, *Leibniz Characteristica Universalis and calculus Ratiocinator* Today, p.1, in http:// www. academia.edu. consulted on Monday 12th of February 2023, 9 A.M.
[117] *Ibid.*, p.2.

3.2. THE THREE LAW OF BOOLEAN ALGEBRA CALCULUS.

3.2.1. The distributive law.

Boole in this law says that the result of an act of election is independent of the grouping or classification of the subject."[118] This means that according to this law, the multiplication or factorization of subjects in bracket is permitted and the result obtain is the same as the given. That is, if we perform the multiplication of two or more variable in a bracket by a given variable the result is the same as performing the addition of each variable in the bracket multiply by the given variable. Example can be seen using the conjunction and disjunction of Boole.

Example using the conjunction; x (u.v) = xu . xv

Example using the disjunction; x(u+v) = xu + xv.

3.2.2. The commutative law.

This law state that, "it is indifferent in what order two successive act of election are perform."[119] Meaning, the order of variables doesn't matter when writing a commutative operation. For example, x and y is the same as writing y and x. It is evident that according to the above law of combination, the order which two symbols are writing is indifferent. The expression xy and yx equally represent the class of things to the several members of which the name or description x and y are together applicable. Hence, we have, xy = yz.

In this manner if x represents "estuaries" and y "rivers", the expressions xy and xy indifferently represent "rivers that are estuaries" or "estuaries that are rivers"[120]

Looking at the given statement, it can be notice that the position of the different terms "rivers and estuary" have not impact the change of each position of the different terms. This is because either you talk of rivers that are estuaries or estuaries that are rivers, the meaning of the statement has not change, but this has kept it meaning and originality.

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[118] George Boole, *Mathematical analysis of logic*, London, Cambridge, Barclay & Macmillan, 1847, p.15.
[119] *Id.*
[120] George Boole, *An investigation of the law of thought*, Cambridge, Macmillan & Co, 1854, p.20.

3.2.3 The absolute identity law.

The law state that, "the result of a given act of election performed twice, or any number of times in succession, is the result of the same act perform once."[121] In other words, as the combination of two literal symbols in the form xy express the whole of that class of object to which the name or qualities represented by x and y are together applicable, it, follow that if the two symbols have exactly the same signification, their combination express no more than either of the symbol taken alone would do.[122] That is, two variables or symbols having the same meaning in common can just be reduce and simplify into one of the meaning in such a way that we remain with single signification of both.

An example is that of saying; "good, good" is the same as to say "good". So "good men" can just be simplify into one as "good men". The simplification of word into only one meaning avoid us to be making repetition because as affirms Boole:

> "*such repetitions of words are indeed sometimes employed to heighten a quality or straighten an affirmation. But this effect is merely secondary and conventional; it is not founded in the intrinsic relation of language and thought. Most of the operations which we observe in nature, or perform ourselves, are of such kind that their effect is augmented by repetition, and this circumstance prepare us to expect the same thing in the language, and even to use repetition when we design to speak with emphasis. But neither in strict reasoning nor in exact discourse is there any just ground for such a practice.*"[123]

This aim of Boole to rendered language strict and exacts will also be Frege objective in his Ideography or Conceptography. Frege in his first part of this book, want to elaborate a new language which will not more be based on ordinary language because such a language is full of ambiguity which render language unprecise and. To Frege, this language must be as he declares: "*a formula language for pure thought. That it is modeled upon the formula language of arithmetic, as I indicated in the title, has to do with fundamental ideas rather than with details of execution.*"[124] This means that, this will be a language based on arithmetic notations which is a discipline which is more rigorous, exact and strict. This is the reason why today logic is called logico-mathematic. In other words, the combination of proposition and numbers.

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[121] George Boole, *Mathematical analysis of logic*, p. 15.
[122] George Boole, *An investigation of the law of thought*, p. 22.
[123] Ibid., p. 23.
[124] Frege Gottlob, *Ideography*, p. 7.

3.3. THE IMPORTANCE OF BOOLEAN EXPRESSION OF SIMPLIFICATION.

3.3.1. Logic and mathematics.

One of the importance of Boolean expression of simplification is the brevity and perspicuity of the symbolic formulas. By brevity and perspicuity, we mean that, the Boolean simplification reduce the use of long sentences or proportions into very short once. That is why Carnap affirms:

> "*a further advantage of using symbols in place of word lies in the brevity and perspicuity of the symbolic formulas. Frequently a sentence that require many lines in a word-language (and whose perspicuity is consequently slight) can be represented symbolically in a line or less. Brevity and perspicuity facilitate manipulation and comparison and inference to an extraordinary degree*."[125]

From this quotation, it is seen that the use of mathematical notation rather than word facilitates the writing and the easy understanding of symbols. This is exactly what the Boolean simplification does when simplifying proposition into algebraic notation or formulas. For example, Paul and Prinder could easily be writing as x*y. this maximise space and reduce the waste of time to be writing everything down.

The importance of Boolean expression of simplification in logic and mathematic can also be seen in the construction of what is known as truth table. Truth table can be defined as: méthode sémantique de validation utilisée en calcul propositionnel consistant à représenter dans un tableau les diverses valeur de vérité que prennent des propositions composées en fonction de celles prises par le propositions simples qui les composent et les particularités logique des connecteurs logiques qui les associent les unes aux autres.[126] Meaning, a truth table show the relationship, in tabular form, between the input values or variable and the result of a specific operator or function on the input variable. As already mention above, the Boolean simplification is made up of binary values 1 which can be express as "true" and 0 which can be represented as "false". The Boolean operators are of three we have:

1) The AND operator which is also known as a Boolean product or conjunction in logic. The Boolean expression xy is equivalent to the expression x*y and is read "x and y" by conjunction. The behavior of this operator is characterized by the truth table below. Table.

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[125] Rudoft Carnap, *Introduction to symbolic logic and its application*, Trans. En. William H. Meyer & John Wilkinson, New York, Dover publication, Inc., 1958, p. 2.
[126] Steven Chapados, *Dictionnaire philosophique et historique de la logique*, Laval, PUL, 2017, p. 466.

X	Y	x * y
0	0	0
0	1	0
1	0	0
1	1	1

From the table above, a rule can be drawn stating that: in the operation AND of the truth table, it is only 1(true) when both variables (x and y) are 1 (true) or 0 (false) when one of the variables is 0 (false).

The OR operator is often referred to as a Boolean sum or disjunction in logic. The expression x + y is read “x or y” by disjunction. The truth table for OR is shown below. Table.

X	Y	x + y
0	0	0
0	1	1
1	0	1
1	1	1

From the table above we can see that this is different from the first truth table thus this will be having a different rule. The rule here is that: it is only 0 (false) when both variables are 0 (false) or 1 (true) when one of the variables is 1 (true).

The NOT operator is often known to as negation. The truth table for not is shown on the table below.

X	~x
0	1
1	0

The rule here is only that once one of the variables is 1 (true) the negation become 0 (false) and this is the same for the 0 (false).

3.3.2. In computer and telecommunication in the Boolean algebra.

The Boolean functions of binary symbols are implemented in computer. As already defined above binary describes a numbering scheme in which there are only two values 0 or 1 and this is the code used in computing systems. These systems use this code too understand operational instruction and user input and to present relevant output to the user. Meaning that computer device function with binary symbols which help them to perform many tasks into the computer system. This also refers to any digital system in which there are exactly two possible states. In digital data memory, storage, processing and communications, the 0 and 1 value are sometime called low and high respectively. In transistors '1' represent a flow of electricity, 0 represent no flow of electricity. The importance of Boolean expression of simplification in computer sciences can be seen into three ways:

- The representation of numbers into the computer using binary code is possible through the form of digital 1s and 0s inside the Central Processing Unit (CPU) and the Random Accessory Memory (RAM). These digital numbers are electrical signals that are either on or off inside the CPU or RAM. Since the binary system uses only two digits which is also called bits and represent numbers using varying patterns of 1s or 0s, it is known as based 2 system. The basis of a system is the number of symbols used by that system. The based two system is the language in which most computer communicated. That is, sending out of information and receiving input. The binary number system is the base of all computing systems and operations. It enables devices to store, access and manipulate all types of information directed to and from the CPU or memory. This makes it possible to develop applications that enable users to do: view websites, create and update document, play game, access of software, and calculation. Apart from this base 2, we have others bases likes base 10 (decimal), 8 (octal), 16 (hexadecimal). An example of how one of this bases (8, 10 or 16) is converted in order to be writing in base system of 2. Let us take base 8 (octal), N_8= 346; this give us 011100110_2. We can also have the subtraction and addition on base 2.

- Binary numbers can be translated into text characters using the ASCII meaning American Standard Code for Information Interchange to store information in the computer's RAM or CPU. ASCII-capable applications, like word processors can read text information from the RAM or CPU. They can also store text information that can then be retrieved by the user at

a later time. For example, the word *hi* is written in the binary system as o110 1000+01101001= 0110100001101001.

- Boolean function or binary are implemented in digital computer circuit called gate. A gate is a device that acts as a building block for digital circuits. They perform basic logical functions that are fundamental to digital circuits. Most electronic devices we use today will have some form of logical gate in them. For example, logic gate can be used in technologies such as smartphone, tablet and computer. The logic gate is based on Boolean algebra. There are six basic logic gates:

We have the three simplest gates which are: the AND gate which is named because, if 0 is called the “false” and 1 is called the “true” the acts in the same way as the logical “and” operator. In the symbol, the input terminals are at left and output terminal is at right.; We have the OR gate which is name from the fact that it behaves after the fashion of the logical inclusive ‘or’. We have the NOT gate use to differentiate it from other types of electronic inverter device. It has only one input and this reverses the logical state if the input is 1, then the output is 0.

In Frege’s eyes, the Boolean logic assume that the formation of concepts through abstraction is the fundamental logical operation and judging and inferring are brought about through direct or indirect comparison of the extension of these concepts. Such a conception which will be develop by Boole in all of his logic, will facilitate and prepared the way for Frege who will come with a new conception. Frege thought that in a complete logic there would also have to be a method of concept-formation that could generate scientifically fruitful concepts with completely new boundaries. That method, he believed, was given in a quantifier notation. The quantifier notation, far from being a minor thing is, in fact, the heart of what made his own logic superior to the Boolean algebra. By means of quantification we can define wholly a new class whose boundaries will not coincide with any part of any previous classes. Frege drew, in this context, attention to his definition of the notions of the continuity of a function, of a limit, and of that of following in a series. Such definitions exemplified for him the advantage of his own logic over Boolean algebra.

3.3.3. The Boolean operations and the electronic research.

Today with the evolution of science and technology especially technology of information and communication, this has enhanced the progress of research in all domains. One of the domains which interest us is that of scientific research in the university. For one to carry

out a scientific work, it is important to master how to make research in an electronic device which can be a Phone or a computer. To have fast and precise information and documents online(internet) we need to insert some syntaxes among these are the Boole logical operations. This include the AND, OR, NOT.

Using the AND in a research help us to narrow our research result and tell the database that ALL the terms must be present in the resulting record. This means that the AND, in a research, play the role of universality "All" by commanding the databases that each and every term used most be include in the result. For example, animal and domestic and herbivores. The example written show that I am searching for animals that are domestic herbivores.

Using the OR in research is connecting two or more similar concept and broaden our results, telling the database that ANY of the research terms can be presented in the resulting record. This means that the OR in a research play the role of Alternative "either …or…". For example. Female or girl or women. The example command for one of the above terms.

Using the NOT in research is to exclude word from our research and narrow our research, telling the database to ignore concept that or not related to what you are searching for. This means that the operation NOT play the role of exception "only". For example, education NOT secondary.

3.4. THE ALGEBRAIC SIMPLIFICATION OF PROPOSITIONS.

3.4.1. Transform proposition into algebra symbols.

Given back the definition of Aristotle on propositions, Boole says a proposition must either be affirmative or negative, and must be also universal or particular. Thus, we reckon in all, four kind of pure categorical propositions.[127]

1st. Universal-affirmative (A), ex. All Xs are Ys. To express the A proposition in the algebraic form this is represented by: $x(1-x) = 0$ or $xy = 1$.

2nd Universal-negative (E), ex. No Xs are Ys. To express the E proposition in its syntax algebraic, this is given by: $xy = 0$.

3rd Particular-affirmative (I), ex. Some Xs are Ys. This is express in algebra as: $xy = 0$.

[127]George Boole, *Mathematical analysis of logic*, p. 19.

4th Particular-negative (O), ex. Some Xs are not Ys. This is express in algebra as: x (1-y) = v.

3.4.2. The binary number of Boole logic

A binary is something based only on two things or parts. In the domain of computer and mathematic a binary number is a system based only on two numbers. This number usually ranges from 0 and 1. This binary number can be seen in the logical simplification of Boole logic. Boole proposes two symbols 0 and 1 which he gave different meanings to him:

> "*in the system of logic, we must assign to the symbol 0 such an interpretation that the class represented by 0y may be identical to 0. A little consideration will show that the symbol 0 represent Nothing. In accordance with a preview definition, we may term Nothing a class. In fact, Nothing and Universe are the two limits of class extension, for they are limits of the possible interpretation of general names, none of which can relate to fewer individuals than are comprised in nothing, or to more than are comprised in the universe.*"[128]

The quotation above demonstrates that the symbol "0" represent universe or universal class represented by the symbol "Nothing" or No. This is a class that enclose the absent of members in a whole. It also represents the truth value "false".

> *N*ow *"let us employ the symbol 1, or unity, to represent the universe, and let us understand it as comprehending every class of objects whether actually existing or not, it being premise that the same individual may be fund in more than one class, inasmuch as it may possess more than in quality in common with other individual"*[129]

From this affirmation, a little consideration will here show that the class represented by 1 must be the universe, since this is the only class in which are found all the individuals that exist in any class[130]. In other word, the symbol "1" represent the universe or universal class which has a positive quantification. When we talk of a positive quantification, we mean that this is the universal positive quantity represented by "All". The number 1 can also represent in logical proposition the truth value "true". Hence the respective interpretations of the symbols 0 and 1 in the system of Logic are *Nothing and universe*.

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[128] George Boole, *An investigation of the law of thought*, p. 33
[129] *Ibid*., p. 34.
[130] *Id.*

3.4.3. The three Boolean operations.

Boole elaborate on the same line of proposition three basic operations in algebra namely: conjunction, disjunction and negation. The conjunction which is a type of propositional connector whose function is to unify two or more propositions. Boole use the sign (.) or (x) to represent **"AND"** which is the conjunction. In the French logic, the conjunction is represented by the sign. The disjunction which is logical connector serve to disjoin two or more propositions. Boole give as sign to represent this to be '+'which use the "**OR**". The disjunction today is represented by the sign V. The negation which in the traditional logic is one of the two types of proposition when classified under quality. This is a proposition which the verb to be is deny from its universality given room to "**No**" or particularity given room to "are not". In the modern logic or symbolic logic, the negation is the negative sign (~) which inverse the truth value of a proposition. The false become true and the true false when each are negated. Boole use the sign (-).

The chapter three already elaborated above put an end to the first part of the dissertation. Throughout this first part, we went over two types of logic: the traditional or formal logic and the modern or mathematical logic. The tradition or formal logic is that logic founded by the logician Aristotle of the antique period and which dominated over two thousand years. The idea of creating a new logic was only recently in the XVII century with Leibniz but this idea was only systematized in the XIX century with one of the greatest logicians Gottlob Frege a German mathematician of training. It is his work on the *ideography* published in 1879 that will constitute the investigation of our part two. Throughout this part, we will elaborate and expose in detail as possible the Frege philosophy in general and his logicism in particular. This will also be divided into three chapters as in part one.

PART TWO: THE FREGEAN REVOLUTION OF LOGIC

The mere invention of this ideography has, it seems to me, advance logic and as I remarked at the beginning, arithmetic was the point of departure for the train of thought that led me to my ideography. And that is why I intend to apply it first of all to that science, attempting to provide a more detailed analysis of the concepts of arithmetic and deeper foundation of its theorems.[131]

[131] Gottlob Frege, *Ideography, a formula language, modeled upon that of arithmetic, for the pure thought*, pp. 7-8.

After spending three chapter in elaborating the pre-fregean foundation of logic which constituted the first part; starting with the Aristotelian traditional logic and it limits. Follow by the modern classical logic with Descartes rejection of traditional logic and the adoption of the four rules, the Port Royal logic and the Kantian heritage. Finally, the tentative of the formalization of logic with Leibniz and Bool. It's time for us in this part two to analyze how with the publication of his first book (Ideography) in 1879, Gottlob Frege revolutionize logic and is been generally regarded as one of the founders of analytic philosophy.[132] This mean that our part two will be the examination of the Conceptography proposed by the mathematician of Inna as the new language. But this investigation on this language can't be done in a whole, that is why we decided to divide this part into three chapters: the weakness and critics of the pre-fregean logic been the chapter four, the philosophical foundation of frege Begriffsschrift our chapter five and last the frege system of logic.

[132] Michael Beaney, *The frege reader*, UK, Blackwell publishing, 1997, Introduction, p. 1.

CHAPTER FOUR: WEAKNESSES AND CRITICS OF PRE-FREGEAN LOGIC.

If one of the natures of philosophy is it aspect of criticism and the idea that science proceed by "*conjectures and refutations*"[133], it is important that before any elaboration of the logicism of Frege, we start by analyzing the different limits been encounter by the pre-fregean philosophers and the critics been done by Frege. This objective will constitute our chapter four. This will be carryout into three sub-chapters; the first sub-chapter is on the question of judgement where the German philosopher rejected the notion of subject-predicate developed in the traditional logic, the rejection of the types of judgments proposed by Aristotle and the last the nature of universal and particular propositions. The second sub-chapter is that of the Begriffsschrift and the war against natural language with the rejection of senseless proposition, sense and refences, and the source of knowledge of mathematics. The last sub-chapter will be new attempt at the foundation of arithmetic with the rejection of psychology, function and concept and number and object.

4.1. ON THE QUESTION OF JUDGMENT.

4.1.1. The rejection of subject-predicate.

The terms subject and predicate play a great and important role in the structure of a preposition and in the form of a syllogism in the traditional logic. As defined by Aristotle a term is that into which the proposition is resolved.[134] This means that a term is what constitutes a premise which is it beginning and end. The premise is made up of two terms: the predicate and that of which it is predicated (subject), whether to be or not to be is added or separated.[135] The disposition of subject-predicate determines the form of deduction in the Stagira logic. We have four forms of syllogism: first, second, third and fourth figure as already developed in part one. For example, No giver is girl. All boys are givers. Therefore, No boys are girls. From this example it seen that the term giver which is the subject of the major premise is also the predicate of the minor premise. This gives us the figure one. That is why the founder of formal logic affirms: "*when one thing is the predicate of another, as of subject, whether things are said of*

[133] Karl Raimund Popper, *Conjectures and refutations the growth of scientific knowledge*, New York, Basic Books, 1962.
[134] Aristotle, *Organon*, p, 82.
[135] *Id*.

the predicate, may be said of the subject"[136]. This means that a term may both be contained in a subject and predicate.

To Frege, the distinction been made in the traditional logic on subject and predicate analysed between sentence goes wrong by following the superficial appearance of natural language too closely. He holds this by given the following argument:

> A *distinction between subject and predicate does not occur in my way of representing a judgment. In order to justify this, I remark that the content of two judgments may differ in two ways: either the consequences derivable from the first, when it is combined with certain other judgment, always follow also from the second, when it is combined with this same judgment or this is not the case. The two preposition "the Greeks defeated the Persians at Plataea" and "the Persians were defeated by the Greek at Plataea" differ in the first way. Even if one can detect a slight difference in meaning, the agreement outweighs it. Now I call that part of the content that is the same in both conceptual contents. Since it alone is of significant for our ideography, we need not introduce any distinction between proposition having the same conceptual content.*[137]

In addition, the problem of multi-generality is an argument that the subject and predicate analysis is incapable of explaining the logical relation of every large class of sentence. This means that, the tradition logic does not admit a large class of sentences where we can find many quantities in a proposition. For example, all boys love some girls. This is a multi-general proposition where one cannot determine the which one is the subject or predicate:

> *In the first draft of my formula language I allowed myself to be misled by the example of ordinary language into constructing judgment out of subject and predicate. But I soon became convinced that this was an obstacle to my specific goal and led only to useless prolixity. we see that there cannot be any question here of subject and predicate in the ordinary sense.*[138]

To overcome this problem, Frege thinks that it is necessary and important to replace the subject-predicate by function and argument.

4.1.2. The rejection of types of judgment.

A judgment is the mental act of asserting by denying or affirming an assertible content. Traditionally, a judgment is said to affirm or to deny a predicate of a subject. As generalized

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[136] *Ibid.*, p. 4.
[137] Gottlob Frege, Idéographie, pp. 16-17.
[138] *Ibid.*, p. 17.

by modern logicians that this become affirmation or denial of a relation among certain terms. The traditional logic distinguishes between: the categorical, the hypothetical and the disjunctive. These three types of proposition have already been developed under the Aristotelian logicism with all the detail.

The author of the *Ideography* thinks that such a distinction made in the traditional logic had nothing to do with logic because the differentiation is mostly taken into consideration the ordinary language. Let us listen to him: "*the distinction between the categorical, hypothetical, and disjunctive judgments seems to me to have only grammatical significant.*"[139] And all what is related to grammar which is link to language is full of ambiguity and this should not be considered in logic which must be free from such propositions. The distinction been accepted and made by the author of *logical writing* is between the apodictic and assertory judgement:

> *The apodictic judgment differs from the assertory in that it suggests the existence of universal judgments from which the proposition can be inferred, while in the case of the assertory one such a suggestion is lacking. By saying that a proposition is necessary I give a hint about the grounds for my judgment. But, since this does not affect the conceptual content of the judgment, the form of the apodictic judgment has no significant for us.* [140]

This means that the only judgment on which our ideography is interested on is the assertory judgment. This assertory judgment is on what all the logic of Frege is based which constitute it entire work. There are two fundamental notion of assertion or assertive force. One notion, the more basic for Frege, is the "logical" notion examined in section 4 of the ideography. The test for deciding whether a given proposition occurs assertively within a given context is the test of exportation: if the proposition can be removed from that context without further ado, on display on a line in a proof by itself, then that proposition is asserted in the original context. The second, different though closely analogous notion of assertion attaches to the speech acts and to their interior, mental counterparts. Here assertion is human action; it is something which a certain person does with a certain proposition at a certain time. The paradigmatic form of an exterior assertion is the uttering of an indicative sentence. As Frege rightly insisted the "assertion sign" of ordinary language, so to speak, lies in the mood of the verb. For instance, I may say firmly and with conviction "someone has just stolen your car" but if I am taking part

[139] *Ibid.*, p. 18.
[140] *Id.*

in an elocution class, or a theatrical performance, I will not be taken to have asserted anything.[141]

4.1.3. Rejection of particular and universal judgment.

The Aristotelian traditional logic usually classifies propositions according to their qualities and quantities. The quality of a proposition is the nature of the proposition of been affirmative or negative. This mean that it is the ability of a judgment to be affirm or deny. While the quantity of a proposition is it nature of extension. Extension here is the ability of a proposition to be universal (no, all) or particular (some or certain): "*categorical statements are distinguished by their quantity, quality and modality. In modality such aa statement is assertoric, necessary or opposite; in quality it is affirmative or negative; and in quantity it is universal (all), particular (in part), singular, or indeterminate."*[142]

According to Frege, such a classification of proposition according to quantity does not need to be envisage. To him: "*the remarks that follow are intend to explain the significance for the purposes of the distinctions we introduce among judgments. We distinguish between universal and particular judgments; this is really not a distinction between judgment.*"[143] If we should not distinguish judgment according to quantity, what then should we distinguish it? He proposes that this should be distinguish in term of content. "*We ought to say "a judgment with a universal content" "a judgment with a particular judgment content". For these properties hold of the content even when it is not advanced as a judgment but as a proposition*."[144] This means that, each time one has to deal with the classifications of judgments in quantity in the logic of Frege, we talk of universal content or particular content.

But what is a content or what is a judgment content? The nature of theses contents can be determined by examining Frege own practice in the Begriffsschrift and the subsequent elaboration of this notion. Frege call "possible judgment content", a proposition stripped of the assertion associated with it in everyday language; moreover, he calls conceptual content this same content of judgment, or part of such a content, stripped of the nuances introduced by everyday language by multiplying synonymous turns. Conceptual content is actually

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[141] David Andrew Bell, *Frege theory of judgment*, doctorate thesis in philosophy, defended in the McMaster university, 1977, pp. 133-134.
[142] Georgios Anagnostopoulos, *A companion to Aristotle*, New York, Blackwell publishing, p. 33.
[143] Gottlob Frege, *Idéographie*, p. 18.
[144] *Id.*

propositional content; ideography is a writing of analyzed propositions, not of ideas in the sense that this term was sometime synonymous with concept.[145]

4.2. THE IDEOGRAPHY AND THE WAR AGAINST NATURAL LANGUAGE.

For most philosophers who write about natural language and meaning, Frege's work has served as a point of departure: to start, and an origin from which to measure divergence along a variety of axes.[146] These axes will be developpe in the following lines.

4.2.1. Rejection of senseless proposition.

Frege by developing the theory of "sense and reference" hold that all preposition must be constitute of a thought and a denotation. This means that if any proposition is asserted, there is always an obvious presumption that the simple or compound proper names used have reference.[147] The principal task of Frege theory become that of ensuring no expression are devoid of sense. In this case senseless propositions are all those propositions that have sense but no reference. Some propositions are senseless in their denotation of object or true value as expose by the author of *sense and references.* For example, the name "Odysseus" has a sense but no references. The fact that some propositions have no references is that, they do not bother with the reference of a part of the sentence; only the sense, not the reference, of the part is relevant to the sense of the whole sentence. The thought remains the same whether 'Odysseus' has a reference or not. [148] Most of such propositions are made by poetics and aesthetics. We can in this sense listen to his affirmation:

> *In hearing an epic poem, for instance, apart from the euphony of the language we interested only in the sense of the sentences and the images and feelings thereby aroused. The question of truth would cause us to abandon aesthetic delight for an attitude of scientific investigation. Hence, it is a matter of no concern to us whether the name "Odysseus", for instance, has reference, so long as we accept the poem as a work of art.*[149]

This means that esthetic and poetics propositions according to Frege are senseless since they do not care about the references of the statements they advance. All what matter to them is the sense they give to their sentences. And this case, the thought loses value for us as soon as

[145] Gottlob Frege, *Les fondement de l'arithmétique*, trad. Fr. Claude Imbert, Paris, Seuil, 1970, pp. 23-24.
[146] Richard Mark Sainsbury, *Departing from Frege*, London & New York, Routledge, 2002, p. 1.
[147] Gottlob Frege, *Écrits logiques et philosophiques*, trad. fr. Claude Imbert, Paris, Seuil, 1971, p. 115.
[148] *Ibid.*, p. 110.
[149] *Id.*

we recognize that the reference of one its parts is missing.[150] It becomes therefore, important that all propositions should have a reference in all languages. As affirm Malherbe:

> "*Frege capital thesis is that the opposition couple sense-reference is applies to all elements of language, to all propositions as well as to their elements: subject and predicate. According to the fregean analysis, all propositions are break down into thought (sense) and a true value (reference).*[151]

In other words, senseless propositions should be rejected in logic due to their imprecision of their references making them to be ambiguous and lose their values. The value of a proposition resides on the present of thought and true value.

4.2.2. Sense and references of a proposition.

Sense and reference are two properties that Frege attribute to all expressions, formula or to all propositions. Sense can be referred in to three principal sense: as sensibility (the five sense of the body), as direction (the way to go somewhere) and as the signification (sense of a phrase). Our main focus will be in the third which is the signification and the signification of a sentence, is that what it means or get, in other word the goal that pursue the one stating this statement.[152] Referent or references is the signification of an object that is what is in reality as it being.

According to Frege, by refences is that which design and which we can call it denotation, the mode of donation of an object. By sense of a proposition, we understand an abstract object which is not a mental content nor a linguistic entity but the objective content of thought expressed in a proposition. This mean that, the content thought of in proposition is the sense of that proposition. So, the sense of a proposition is the thought which it contained. Frege rejected the reference to be the content of a proposition in that if a word is substitute by another with the same reference although having different sense, this can't have any influence on the reference of the proposition. But we notice that the thought undergoes some modification. For example, "the stars of the morning is a body shine by the sun" is different from the content thought in "the star of the evening is a body shine by the sun". One could hold one of these propositions to be true and the other false if ignore that the star in the morning is the same in the evening. [153]

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[150] Peter Geach & Max Black, *Translation from the philosophical writing of Gottlob Frege*, Oxford, Basil Blackwell, 1960, p. 63.
[151] Jean François Malherbe, *Épistémologie Anglo-saxonne*s, Namur, Presse Universitaire de Namur, 1981, p. 29.
[152] André Comte-Sponville, *Op.cit.*, p. 831.
[153] Gottlob Frege, *Ecrits logiques et philosophiques*, p. 108.

From the example above, we see that the two propositions have the same referent but different senses. As it is possible to have propositions of the same senses but different references.

But, "is it possible that a sentence as a whole has only sense, but no references? Frege answer to this question by saying:

> *At any rate, one might expect that such sentence occurs, just as there are parts of sentence having sense but no references. And sentences which contain proper names without reference will be of this kind. The sentence 'Odysseus was set ashore at Ithaca while sound asleep' Odysseus has a sense. But the name Odysseus occurring therein has no references.*[154]

To such propositions having sense with no reference what then can its reference be since for Frege, the task of logic is to ensure that no expression is devoid of a reference[155]. Frege thing that for such propositions the only solution that we are left with is the truth value. The truth value of all sentences with sense but no references becomes it denotation: "*we are therefore driven into accepting the truth value of a sentence as constituting its reference. By the truth value of a sentence I understand the circumstance that it is true or false. There are no further truth values."*[156] He continuous by affirming:

> *If our supposition that the reference of a proposition is its truth value is correct, the latter must remain unchanged when a part of the sentence is replaced by an expression having the same reference. And if the truth value of a sentences is its reference, then on the one hand all true sentences have the same references and so, on the other hand do all false sentences*.[157]

From the following analysis, it resorts that when talking of the question of sense and reference of a proposition, this should be considered into two ways; sense and refences of a sentence in term of the direct relation between though and object or reality. Where a sense has it inference as object represented in the reality. The other is the sense and reference of a sentence in term of thought and truth value. Where the sense has no object of representation. He reduces this true value into true and false.

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[154] Peter Geach & Max Black, *Op.cit.,* p. 62
[155] This is our translation of « *pour Frege, la tâche de la logique est de veiller à ce qu'aucune expression ne soit dépourvue de référent* » Jean François Malherbe, *Op.cit.,* p. 23.
[156] Gottlob Frege, *Ecrits logiques et philosophiques,* p. 110.
[157] *Ibid.,* p. 111.

4.2.3. Source of knowledge of Arithmetic.

Frege source of knowledge of arithmetic is one of the theories which he developed in his book: *The foundation of arithmetic* published in 1884. In this book, the Jena mathematician investigates the philosophical foundation of arithmetic defined here as, the science of entire numbers, of their properties and relation.[158]These numbers which were not only developed by Frege but also by his predecessors who gave different foundation on its knowledge and which were been rejected and criticized by the founder of analytical philosophy. The different critics been done by Gottlob Frege concern the conception of the philosophical foundation of arithmetic by author such as Kant, Mill, Leibniz and Weierstrass. The various criticism advances by our author are as follow:

> *On comprendre mieux l'actualité d'une réfutation de Mill si l'on pense que la philosophie de l'arithmétique réduit en 1894 des arguments directement inspirés par l'empirisme anglo-saxon. Chez Hankel, Frege critique la confusion entre signe et désigné, et d'explorer le champ entier de l'arithmétique sur le modèle d'un usage protocolaire des signes. […] A Kant est reproché de l'appel à l'intuition, serait-elle intuition pure, et une pensée trop asservie au langage commun et à la théorie classique. C'est-à-dire aristotélicienne de l'abstraction. Enfin, des mathématiciens aussi avertis que Leibniz et Weierstrass s'accordent pour définir les nombres particuliers au moyen de l'unité et de l'adjonction de l'unité. Encore faudrait-il savoir ce qui est unité, jusqu'alors confondue avec un objet quelconque, support occasion de l'acte de compte. La célèbre définition de Leibniz repose en réalité sur l'intuition du bouclier, guère différent des nonnettes et tas de cailloux de Mill. Ill n'est pas étonnant que ce dernier ait repris textuellement la définition de Leibniz.* [159]

According to Frege, the philosophical foundation of arithmetic is neither the intuition as affirm Kant, nor the empiricism as hold Mill but this is the a prior: "*I hope I may claim in the present work to have made it probable that the laws of arithmetic are analytic judgment and consequently a priori. Arithmetic thus becomes simple and development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one.*"[160] Analytic judgments are those arguments that do not depend on the observation for its truth. This is a pure construction of the mind independent of the subject or individual.

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[158] André Lalande, *Op.cit.*, p. 79.
[159] Gottlob Frege, *Les fondements de l'arithmétique*, p. 28.
[160] Gottlob Frege, *The foundations of arithmetic (1884)*, trans. en. J. L. Austin, New York, First Harper TorchBook, 1960, p. 99.

4.3. A NEW ATTEMPT AT A FOUNDATION OF ARITHMETIC.

4.3.1. The rejection of psychology.

Our first task is to define psychology, that is to say, to delineate the subject matter of the field, and to describe the methodology it characteristically employs. Like many disciplines, psychology has its origin with the ancient Greeks. Plato and Aristotle, for example, about the fourth century B.C, wrote treatises on human psychology which they regarded as the study of the soul which refers to its etymology *psyche* meaning "soul" and *logos* meaning discourse or science. "*Today, psychology is usually defined as the science of the human behavior*."[161] By science, we mean that, the method of psychology must be objective and experimental. The subject of psychology in logic, this is the view based on the idea that the law and rule of logic are reducible to psychological or thought laws. Psychology view logic from the angle of psychic, mental or subjective acts by which logical operations and calculations are carried out, assuming that human psychology forms the bedrock on which all principles are based. This discipline is a variant of reductionist naturalism, which reduces everything to natural phenomena (psychological type). [162] Frege does not share such a definition, to him: "*one has to separate sharply the psychological from the logical, the subjective from the objective*" [163] But what does it mean? And why is Frege holding such a view?

This means that there must be a distinction to an extern a rejection of psychology in logic because these two disciplines are not the same and never will they converge each other such that laws of logic will depend on the psychology of individual. From their various subject matters, the two greatly differ. According to Frege the subject matter of logic which is thought and judgment has only one task to establish and explained the law of valid inferences. This has nothing to do with products of mental processes which is the subject matter of psychology based on intuition, instinct and representation that cannot be described as true or false at all. Even when psychology is concerned with thought and judgment this is not in term of their justification but their causes which are capable of leading us in to errors. We can in this sense listen to Frege:

> *The subject matter of logic is therefore such as cannot be perceived by the senses and in this respect, it compares with that of psychology and contrast with that of natural sciences. Instinct, ideas etc. are also*

[161] Thompson & Debold, Psychology: *a systematic introduction*, New York, McGraw-Hill, 1971, p. 4.
[162] Steeven Chapados, *Op.cit.*, p. 337.
[163] Gottlob Frege, *Les fondement de l'arithmétique*, p. 122.

> *neither visible nor tangible. All the same there is a sharp divide between these disciplines*[164].

This distinction or rejection of psychology from logic can be seen further in that psychology is subjective while logic is objective. Subjective to Frege refers to something that is mental, to an idea, to something that is 'in us. Meaning that psychology is what depend on the individual which is not intersubjectively accessible to everyone. Example, our sensation, mental process, sense expressions, ideas, understood in the psychological sense are subjective because they are like images or sensation which we cannot know to agree with anyone else's. Is objective to Frege what is exactly the same for all rational beings, for all who are capable of grasping it. This does not depend on any subjectivity but is intersubjectively accessible. The true that I have is the same true you can have following a certain laws or rules of inferences. Therefore, no less essential for mathematics than the refusal of all assistance from the direction of psychology, is the recognition of it close connexon with logic.[165]

4.3.2. Function and concept.

To define what we understand by the word function, Frege start from the notion of mathematic and which he will later extend it. In mathematics, we understand by 'function of x, is taken to be a mathematical expression containing x, a formula or figure containing the letter x.[166] A function become in this sense, a special type of a relation, one that associate each element of the domain with a unique element of range. Of course, a given element in the domain might be associated with more than one element of the range.[167]

So, the expression;

$2.x^3 + x$ and $2.23 + 2$ is a function of x and 2 respectively.

But Frege is not satisfied with such a definition because it does not distinguish between the content and form, sign and thing signified; a mistake mostly committed by the contemporary mathematicians. These limits drive Frege to a more precise interpretation of function. We call x the argument of the function and recognize in:

$2.1^3 + 1$,
$2.4^3 + 4$,
$2.5^3 + 5$.

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[164] Carl Wolfgang, *Frege theory of sense and references*, Cambridge, Cambridge University Press, 1994, p. 28.
[165] Gottlob Frege, *The Foundation of arithmetic*, introduction.
[166] Gottlob Frege, *Écrits logiques et philosophiques*, p. 81.
[167] Richard Mendelson, *The Philosophy of Gottlob Frege*, Cambridge, Cambridge University Press, 2005, p. 8.

The same function with different argument, viz. 1, 4, and 5. From this, we may discern that the proper essence of function resides common element of its expression; That is, what is present in

$2.x^3 + x$

when we omit the letter "x". We could write this somewhat as follows:

$2.()^3 + ()^{168}$

Frege affirms in this sense:

> *I am concerned to show that the argument does not belong with the function, but goes together with the function to make up a complete whole; for the function by itself must be called incomplete, in need of supplementation, or unsaturated. What we obtained by completing function with argument is what we called "the value of a function for an argument. In this sense, 3 is the value of the function* $2x^2 + x$ *for the argument 1 because* $2.1^2 + 1 = 3$.[169]

Frege after this interpretation of function, extend into two directions the notion of function:

> *We can distinguish two directions in which this has happened. In the first place, the field of mathematical operation, and that serve for the constructing functions has been added. Secondly, the field of possible arguments and values for functions has been extended by the admission of complex number*.[170]

The different mathematical operations that have been added in the existing operation apart of the addition (+), multiplication (.) and exponential ($\underline{(\ )^x}$) are: equality (=), greater than (>), and less than (<), in such a way that we can write $x^2=1$ where x represent the argument. We can determine the value of this function for the different argument by replacing it successively by the number -1, 0, 1, 2 and obtain the following,

$(-1)^2 = 1$
$0^2 = 1$
$1^2 = 1$
$2^2 = 1$

Among these equations, the first and the third are said to be true and the rest false. This is because once we multiply the (-1) by itself we obtain a positive number 1 and in the third (1) multiply by itself is 1. The multiplication of the rest of the two by itself can't give us 1. The value of the function is therefore the truth value and we distinguish two truth values; 'the true

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[168] Peter Geach & Max Black, *Op.cit.*, p. 24.
[169] Gottlob Frege, *Écrits logiques et philosophiques*, pp. 84 - 85.
[170] *Ibid.*, p. 88.

and the False'. For example, "2^2 =4", "2>1", "$2^4 = 4^2$" is true. And (2^2= 4) = (2>) is correct. To Frege the use of the =, <, and > in this addition is due to the fact that mathematical laws are develop by the laws of logic.[171]

Frege continues his second direction by the arguments and values for functions by dividing the statement into two parts; none complete of itself and the other in need of supplementation or unsaturated. The example given by Frege is that we can split the sentence

'Caesar conquered Gaul'

Into 'Caesar' and conquered Gaul'. The first part is part is 'Caesar' which is the proper name and is the part complete in itself which represent the argument. The second part 'conquered Gaul' is unsaturated. It contains an empty place which can only have a complete sense when the place is fill up with a proper name. This unsaturated part stand for the function. Frege hold that we must considered all objects without restriction as values of functions. He defined an object as anything that is not a function, so that the expression for it does not contain any empty space. Example,

'The capital of the German Empire'
When we split up to the parts we obtain,
'the capital of' and 'the German Empire'

The first part is unsaturated and is the expression of a function while the second part is complete on itself and is the argument. [172]

4.3.3. Numbers and Objects.

According to Frege, an object is any sentence that expresses a singular thought consists of an expression is called an object. This means that an object is a proper name, a general term or a definite article use as the subject in a proposition: "*on the other hand, a name of an object, a proper name, is quite incapable of been used as a grammatical predicate*."[173] From the quotation, proper names which can be attribute to object can only be used as a subject never as a predicate. For example, "the morning star is Venus", we have two proper names, "morning

[171] These examples concerning the first direction is taken from Gottlob Frege, *Écrits logiques et philosophiques*, pp. 88-89.
[172] Peter Geach & Max Black, *Op.cit.*, pp. 31-32.
[173] *Ibid.*, p. 43.

star" and "Venus" for the same object. In the sentence "the morning star is planet" we have a proper name, the 'the morning star'.[174]

Concerning a number, Frege first of all defined a number as an extension of a concept: "*similarly we could say I belong to a concept F, if the proposition a does not fall under F is not true universally, whatever a may be , and if from the proposition "a falls under F" and "b falls under F"*"[175] This definition of a number will later be given in a general way as "*the number (n + i) belong to a concept F, if there is an object a falling under F, and such that the number n belong to the concept "falling under F, but not a*".[176] But after a perfect examination, he fully realized imperfection and drawbacks of the idea and pursued his investigation on further one. The next definition given by Frege which seems to him to be a perfect one is that of a number defined with relation to equinumerosity of an object. In this sense, he gave the following assertion:

> *Our first attempt broke down, because we had defined only the predicate which we said was asserted of the concept, but had not given separated definitions of 0 and 1, which are only elements in such predicates. This resulted in our being unable to prove the identity of numbers. It became clear that the number studied by arithmetic must be conceived not as a dependent attribute, as substantivally. Number thus emerged as an object that can be recognized again, although not a physical or even a merely spatial object, nor yet as one of which we can form picture by means of our imagination. [177]*

From the assertion above, the teacher of Jena demonstrate that the relationship that exist between any number and object is based on the fact that every individual number is a self-subsistent object: "*every individual number manifest itself as an independent object in that this constitute just one part of the assertion done on the concept. […]. This independence is manifest in all arithmetic, for example in the equation 1+1=2*."[178]

In this equinumerous of numbers and objects, Frege expresses the correlation that exist between these two concepts. Through this correlation, he rejects the view of his predecessors and contemporaries who hold that arithmetic is abstract and this does not have something with

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[174] *Ibid.*, p. 44.
[175] Frege Gottlob, *The foundation of arithmetic,* p. 67.
[176] *Id.*
[177] *Ibid.*, p. 115-116.
[178] This is our translation of « *chaque nombre particulier se manifeste comme un objet indépendant en ceci justement qu'il constitue seulement une partie de l'assertion porte sur le concept. […]. Cette indépendant est manifeste dans toute l'arithmétique, par exemple dans l'équation 1+1=2.* » in Frege Gottlob, *Les fondements de l'arithmétique*, p. 184.

our external world. In the same way, he rejects the view of those who taught that numbers depend on the subject: number is neither a collection of things nor a property of such, yet at the same time is not a subjective product of the mental processes either, we concluded that s statement of number asserts something objective of a concept.[179]

Finally, this chapter four emphasize on the elaboration of the critics been made by Frege to the traditional logic particularly that of Aristotle and the ordinary language. It has been seen that, Frege criticizes the formal logic due to its attachment to ordinary language. This attachment to natural language has made logic to lose all its rigorous character or aspect. The absent of rigorous is due to the fact that natural language is ambiguous and polysemous making the language not precise. There is the present of psychology in the ordinary language which was also been used in logic by integrating the aspect of subjective than objective. To eradicate all these limits of logic and ambiguous of ordinary language, Frege puts forward a new system of logic founded on some notions of arithmetic.

[179] Frege Gottlob, *The foundation of arithmetic*, p. 115.

CHAPTER FIVE: PHILOSOPHICAL FOUNDATION OF FREGE CONCEPTOGRAPHY

As already enunciated in the partial conclusion above, this chapter will be the analysis of Frege foundation of ideography. To well elaborate his logico-mathematics, Frege start by putting into place certain foundation on which all his logic will be based. To well interpretate these foundations, we sub-dived this chapter into three parts: the first will consist of bringing out the different symbols, laws and role in which Frege's Begriffsschrift are based. The second will be follow by the eight axioms on which the different inferences are derive. The last is to put forward the traditional logic and Frege logic to analyze their differences.

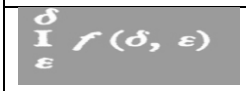
5.1. FUNCTION AND CONDITIONALITY OF BEGRIFFSSCHRIFT.

Frege refers to some uses in his Begriffsschrift a syntax which is made up of combination of capital and lower alphabet of Greek letters, Gothic letters, Germany letters, Latin or Roman letter. There is also the presence of: strokes, lines, symbols, signs, parentheses, formula, points and commas. He gave out some rule of the ideography and axioms of inferences in which his logic will be founded.

5.1.1 Analysis of main symbols.

Symbols	**Description**
⊢———	Judgement stroke
———————	Content strokes
A, B, Γ, Δ	Greek uppercase letters (staring with…) function as the abbreviation for any type of content
	Conditional stroke
———————————	Inference lines
(X): (XX): :	Scheme indicating the abbreviation of a judgment in an inference

═══════	Line of inference indicating that one of the steps have been abbreviate
¬⊤	The negation stroke
(…≡…)	Sign of identity stroke
(…)	Parenthesis.
Φ, X, **Ψ**	Sequence of Greek uppercase letters starting with Φ
	Sign of generality: hollow with a lowercase Gothic letter in the inside.
ɑ, b, c…h	Sequence of lowercase Latin letters beginning with… expressing generality.
Λ, M, N, P, Σ	Uppercase Greek letters.
… | …	Table of substitution: the signs found on the left of the stroke are substitute by those found on the right.
X, y, z, …	Sequence of Latin lowercase beginning by …
‖…….	Definition stroke
F (…)	Latin uppercase letter with parenthesis. Signify a property according to the formula 69, p.75.
F (…,…)	Latin lowercase letter with parenthesis. Signify a relation according to the formula 69, p. 75.
α, β, η. Σ	Sequence of Greek lowercase letter (starting with α) appears in the abbreviated formula.
$\begin{cases} F(a) \\ f(\sigma.\alpha) \end{cases}$	Abbreviated formula representing heredity, which expresses the fact that the property *F* is hereditary in sequence determined by *f*.

	Sign of generality: hollow with a Gothic uppercase letter inside. (sequence starting with ∀)
$f(x_{\gamma}.y_{\beta})$	Sequence of the Latin lowercase letters (starting with *x*) with Greek lowercase letters as an index serving to indicate an argument place.
$\frac{\gamma}{\beta} f(x_{\gamma}.y_{\beta})$	Abbreviated formula representing the succession, which express the fact that *y* follow *x* in the sequence determined by *f*.
$\frac{\gamma}{\beta} f(x_{\gamma}.z_{\beta})$	Abbreviated formula representing membership in a sequence, which express the fact that *z* belongs to the sequence determined by *f* starting with *x*.
M	Latin uppercase letter.
	Abbreviated formula representing the univocity of a procedure, which express the fact that, the procedure f is univocal.

5.1.2. Rules of ideography.

Frege ideography is made up of several rules[180] that guide it Conceptography. These rules are:

R 1: In a statement, all or some occurrences of a Latin letter can be replaced by a Latin function marker of the same type. Meaning, that in a statement letter can be change to function but this must be of the same nature. Example. Let t1 be a term, t2 a function name of the type T and I a Latin letter of type T which figure in t.

$$\frac{\vdash t}{\vdash \text{replace}(t_1, l, t_2).}$$

R 2: In the super-constituent of a statement, a Latin letter may be replaced by a Gothic letter which does not appear there, and this same Gothic letter placed above the hollow

[180] The rules of the ideography been established here are taken from the thesis of Meven CADET, *Les Grundgesetze der arithmetik de Frege Idéographie: genèse, syntaxe, sémantique*, defended at Université de Paris 1 Pantheon-Sorbonne, 2017, p. 412.

quantification before the super-constituent, provided that the Latin letter does not appear in any sub-constituent of the statement.

R 3: In a statement, all occurrences of a Gothic letter can be replaced by the occurrences of another Gothic letter of the same type, provided that the second does not appear on the hollow above which the first appears. Let t be a term of the ideography, I and I^i two Gothic letters of the same types as appear in t and that no occurrences of I^i does not appear in the scope of I.

$$\frac{\vdash^t}{\vdash^t [I'/I]}$$

R 4: In a statement, all occurrences of a lowercase Greek letter can be replaced by occurrences of another lowercase Greek letter, provided that the second does not appear in the scope of the gentle spirit in which the first appears. Let t be a term of the ideography, I and I^i two lowercase Greek letters as shown in t and no occurrence of I^i does not appear in t in the range of I.

$$\frac{\vdash^t}{\vdash^t [I'/I]}$$

R 5: In a statement, two successive occurrences of a horizontal lines can be merged, that is, replaced by a single line.

$$\frac{\vdash^t}{\vdash^t [\text{‘— —’}/\text{‘—’}]}$$

R 6: Two sub-constituent of any statement can be interchange.

R 7: If any sub-constituent appears twice in a statement, one of the two occurrences can be delated.

R 8: Any sub-constituent of a statement can be swapped with the super-constituent if they are both preceded by a negation.

R 9: Two successive negation symbols can be added or omitted in front of the constituent or any sub-constituent.

R 10: Given two statements such that the second correspond to a sub-constituent of the first to which has been added the judgment symbol. We can deduce a third statement identical to the first unless in that the sub-constituent in question has been delated.

R 11: Given two statements such that the super-constituent of the first appears as a sub-constituent of the second. We can deduce a third statement sharing the super-constituent of the second, and whose sub- constituent are the sub-constituents of the first and the second, except for the term in question.

R 12: Given two statements which share their super-constituent and in which is sub-constituent of one appears, once preceded by the negation symbol, as a sub-constituent of the other. We can deduce a third statement which has the same sur-constituent and whose sub-constituents of the first and the second, except for the term in question.

5.1.3. The roles of symbols.

The roles of the different symbols been represented above will be discourse on this sub-chapter. It should be noted that not all the role of the symbols will be given since Frege himself did not gave all their roles in his ideography.

The judgment: According to frege, a judgment is the progression from the thought to it references or truth value. This means that, a judgment is the relation between the idea or sense to it object or denotation or truth value. Frege holds that a judgement must always be represented in his logic by means of the sign: ⊢——A

The sign of the judgment is made up of two strokes: let us call the horizontal stroke the content stroke and the vertical stroke the judgment stroke. If we omit the vertical stroke at the left of the end of the horizontal one, the judgment will be transformed into a mere combination of ideas. We express ourselves paragraphically, using the word “the circumstance that” or “the proposition that”. The content stroke will in general serve to relate any sign to the totality of the sign that follow the stroke. [181] The modern representation of the judgment is given as: **P(a)** =1

[181] Gottlob Frege, *Idéographie*, pp. 11-12.

The conditionality: The conditionality is Frege's proposition of the hypothetical or implication which is expressed by the terms "if…then…" To Frege, if A and B denote judgeable content, then three are four possible possibilities.

1. A is affirmed and be B affirm.
2. A is affirmed and B is denied.
3. A is denied and B is affirmed.
4. A is denied and B is denied.

To represent the conditionality, Frege uses the sign:

This stand for the judgement that the third of these possibilities dos not take place, but one of the three others does. To read this conditionality, the antecedent is place at the bottom of the sign and the consequence at the top of the sign. In this sense, the sign above could be read as "A is the necessary condition of B or B is the necessary consequence of A". The symbol for the representation of the conditionality also known as the hypothetical or implication is given as: **(A→B)** or **(A⊃B)**.

Concerning the truth value of the conditionality, once this is denied, this means the third possibility takes place, hence that A is denied and B is affirmed.

Of the case in which the conditionality is affirmed, the following three judgment can be made: 1. A must be affirmed and B does not matter. From here, we have two possibilities; the first and second. 2. B has to be denied then is immaterial (does not matter). Here we have the second and the fourth possibilities.[182]

Negation: the third notion of the Conceptography introduced by Frege is the negation stroke: "if a short vertical is attached below the content stroke, this will express the circumstance that the content does not take place. For example, ├──A means "A does not take place". I can call this short vertical stroke the negation stroke."[183] The negation stroke is unary function symbol that attaches to names, terms, concept, and give them an opposite truth value. This means the negation stroke transform any proposition into its false nature. As already represent above, the symbol for negation is given by Frege as: ──┬──**A.** the part of the horizontal stroke to the right of the negation stroke is the content stroke of A; the part to the left of the

[182] Gottlob Frege, *Idéographie*, pp. 19-20.
[183] Gottlob Frege, *Ideography*, p. 17.

negation stroke is the content stroke of the negation of A. The modern representation of the negation stroke is symbolise as: **¬A or ~ A**

Identity of content; frege start in this symbol by given some enlightenment concerning the differences that exist between identity, conditionality and negation. He affirms:

> *Identity of content differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses only a relation between their respective contents, they suddenly displace their own selves when they are combined by means of the sign for identity of content: for it expresses the circumstances that two names have the same content*."[184]

The symbol used by Frege to represent this is **(≡)**. This means that the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely. The example of a direct identity can be found in his theory of the sense and reference development in his article present on his book known as "*philosophical writing*". But it is important to note that the need for a sign for identity of content rests upon the following consideration: the same content can be completely determined in different ways; but that in a particular case two ways of determining it really yield the same result is the content of a judgment.[185] The modern representation of the identity has not change, this is steal the same.

Generality: this what is also known as the concavity or quantification. Due to the limits been face by the traditional logic, Frege proposes another way what he called concavity:

> *In the expression of a judgment we can always regard the combination of sign to the right of judgment as a function of one of the signs occurring in it. If we replace this argument by a German letter and if in the content stroke we introduce a concavity with this German letter, this stand for the judgment that, whatever we take for its argument, the function is a fact or generality (concavity)*."[186]

This means that, the generality is a form of the transformation of the judgment stroke through the modification of some part of this judgment stroke. This can be represented in the symbol below:

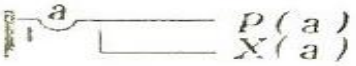
[184] *Ibid.*, p. 20.
[185] *Ibid.*, p. 21.
[186] Gottlob Frege, *Idéographie*, p. 33.

The horizontal stroke to the left of the concavity is the content stroke for the circumstance that, whatever we may put in place of ***Φ(a)***, holds; the horizontal stroke to the right of the concavity is the content stroke of ***Φ(a)***, and her we must imagine that something definite has been substituted for *a*.[187] The modern symbolization of generality is: **∀x(H(x) → B(x))** for the universal propositions and **∃x(H(x) ∧ B(x)**) for existential propositions (particular). Example of generality is to represent the four propositions elaborated by Aristotle.

A: Universal Affirmative 

E: Universal Negative 

I: Particular Affirmative. 

O: Particular negative 

5.2. AXIOMS AND THEIR INFERENCES.

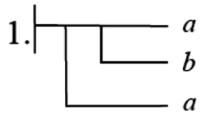
Frege in his ideography came out with nine axioms or principles on which all his one hundred and thirty-three (133) inferences will be base. In fact, "*Frege's claim that every truth of arithmetic is provable from purely logical principles was to have been demonstrated by the proof, from purely logical principles, of a small core of arithmetical truths. That core in turn, it was supposed, word suffice for the proofs of every arithmetical truth*."[188] These cores known as axioms[189] are divided into four: three for the conditionality (1,2 and 8), three for the negation (28, 31and 41), two for the identity (52 and 54) and one for the generality (58).

5.2.1. On conditionality

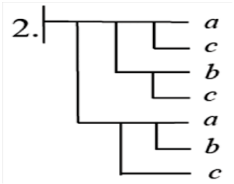
A 1: This axiom holds that "*if a proposition **a** hold, then it also holds in case arbitrary proposition **b** hold.*". This means that, in the case that *a* is true, this automatically means that it is true when ***b*** is true. An illustration can be this, taken ***a*** as nature is constitute of living things

[187] *Ibid.*, p. 34
[188] Philip Ebert, & Marcus Rossberg, *Essay on Frege's Basic Law of Arithmetic,* Oxford, Oxford University Press, 2019, p. 44.
[189] An axiom in mathematic or logic are unprovable arguments, statements, rules or first principle accepted as true because these are self-evident or particularly useful. Frege uses these axioms in order to lay down most of his inferences which are discoursed in his book Begriffsschrift. The axioms are analysed in the second part of this Begriffsschrift, pp. 29-54.

and ***b*** living organizing are things. If nature is constituted of living things, this also hold in case the living organizing are things. This axiom can be represented to Frege as:

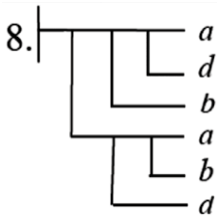


The modern representation can be: ⊦**(b∧a) →a**. **A 2:** This axiom state that: "*if a proposition **a** is a necessary consequence of two proposition **b** and **c** an if one of these **b**, is in turn a necessary consequence of the other, **c**, the proposition **a** is necessary consequence of the later one, **c** alone.*" Frege represent this in his ideography as:



This can be represented today as: ⊦ **((b ∧ c) →a) → ((b → c) → (c → a)**

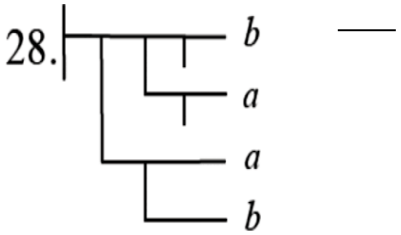
A 8: This axiom holds that: "*if two condition have* a *proposition as a consequence, their order is immaterial (indifferent)"*



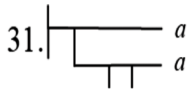
This can be read today as: ⊦ **((d ∧ b) →a) → ((b ∧ d) →a)**

5.2.2. On negation.

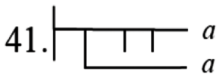
A 28: On this axiom, "*we cannot at the same time affirm **a** and deny **a***". Meaning, the case in which a judgment is denied and a judgment is affirmed does not take place. For example; taken ***b*** as man is alive and ***a*** as man breathes. We have "if man is alive, his breath can be inferred, then from the circumstance that he does not breathe his death can be inferred". The modern representation can be written as: ⊦**(b→a) →(¬a→¬b)**



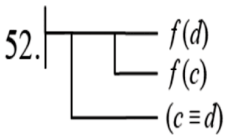
A 31: Here, "*double negation is affirmation or denial of the denial is affirmation*" This is given today as: ⊦¬ ¬**a → a**.



A 41: On this axiom "*the affirmation of a condition denies the denial of the condition*" This means that, the affirmation of ***a*** denies the denial of ***a***." This is written today as: ⊦**a→ ¬ ¬a**



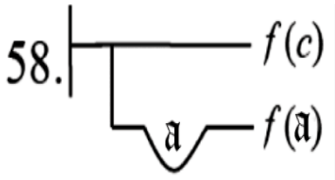
5.2.3. On identity content and generality.

A 52: On this axiom, "*the case in which the content of* ***c*** *is identical with the content of* ***d****, we could everywhere put d for* ***c.*** 

This is given as: ⊦**(c≡d) →(*f*(d)→*f*(c))**

A 54: Here, "the content of ***c*** is identical to the content of ***c***" ⊢——— (c≡c). The modern representation is: ⊦**c≡c**

A 58: The axiom holds that, "the generality ***f(a)*** takes place, whatever we may understand by ***a***. If therefore the generality… ***f(a)*** is affirmed, *f(c)* cannot be denied.



This is given today as: ⊦**(∀*a*) *f*(*a*)→*f*(*c*)**.

5.3. DEMARCATION BETWEEN IDEOGRAPHY AND THE ARISTOTELIAN TRADITIONAL LOGIC.

It is clear that there is a sharp distinction between Frege and Aristotle logic since on is of the traditional logic and the other is of the modern logic. This distinction will not be on what we will based our own due to the fact such a difference is already known by all. We will focus our on the methodology, technic and definition.

5.3.1. On the methodological plan.

When we talk of the methodology, we refer to the way in which the various inferences are been inferred in both system of logic. The Begriffsschrift is distinct from the Aristotelian traditional logic in that it gives way to the reformulation of the several moods of inferences

present in the in the organon. In fact, there exist in the organon four figures of reasoning mostly known as syllogism and these four syllogisms are made up of sixty-four possible moods. We usually recognize in these sixty-four moods nineteen that are valid. These nineteen are possible through the various combinations of propositions present in its figure. For example;

No girls are givers.

Some children are girls.

Therefore, some children are givers. Mood: **EIO**, figure 1.

The table of the nineteen valid propositions can be seen on page twenty (20) above.

Contrary to Aristotle, Frege system of logic use one mood of inference: "*the modus ponens*" or the rule of detachment. The modus ponens is an abbreviation of the Latin modus ponendo ponens which signify affirming affirmed. This means that, the modus ponens is a hypothetical reasoning or syllogism where by the minus premise is affirming and the conclusion itself is affirms. This is a logical mood of reasoning which consist of affirming an implication by it antecedent and its consequence. For example;

If Paul is a boy (A) then he is a mal. (B)

Paul is a boy. (A)

Therefore, he is a mal. (B). This can be represented in the following scheme:

$$\frac{A \longrightarrow B \quad A}{B}$$

According to frege therefore:

> *The figures and moods of Aristotelian syllogism are prolix and their prolixity need to be reduced so that the conclusive moods appears clearly and that the letter are distinct from the inconclive moods; it is therefore necessary to reduce and rectify. Remembering what is required for the conduct of the tests requires tis double operation. Frege proceeds into two steps: he reduces the moods of syllogistic inference to the modus ponens; he reformulates in his in his notion the conclusive syllogisms by adding the conditions like to render them conclusive."*[190]

[190] This is our translation of « *les figures et les modes du syllogisme aristotélicien sont prolixes et leur prolixité demande à être réduite pour qu'apparaissent nettement les modes conclusifs et que ces deniers soient distincts des modes non conclusifs ; il faut donc et réduire et rectifier. Retenir ce qui est requis à la conduite des preuves demande cette double opération. Frege procède en deux étapes : il réduit les modes d'inférence syllogistique au modus ponens ; il reformule dans sa notion les syllogismes non conclusifs en ajoutant les conditions susceptibles de les rendre conclusifs*. », Ali Benmakhlouf, *Frege le nécessaire et le superflus*, Paris, Vrin, 2002, pp. 79-80.

The modus ponens becomes in this sense the excellence mode of inference according to Frege because: "*the mixture of the conclusive moods and the nonconclusive moods in the figure of Aristotelian syllogism produce the same effect as the mixture within logic and psychology: it make loose, for those of deductions, between the necessary and superfluous.*"[191] It is important to note that the modus ponens differ from the modus tollens, which is the contrary of the modus ponens. Here, this is the implicational reasoning where the minor premise denies and the conclusion also denies. Both modus ponens or tollens can be invalid in this becomes modus ponens tollens for the modus ponens and modus tollens ponens for the modus tollens. For example.

Valid modus:	1 A ⟶ B	2 A ⟶ B	Invalid modus:	3 A ⟶ B	4 A⟶ B
	A	~B		B	A
	B	~A		A	B

5.3.2. On the technical plan.

Another way of distinguishing between Frege and Aristotelian system of logic is looking at their technical plan. Once we talk of a technical plan, many questions start rising our mind. The technical plan here refers to the structure of a proposition. The Aristotelian syntax logic is composed of concept and class. This is a bivalent logic which is made up of two truth values: true and false subjugated to the gramma of ordinary language through two terms subject and predicate. This means that, the relation of subject and predicate in Aristotelian logic is attached to grammar of natural language. This relation to Frege is superfluous:

> *This relation (subject and predicate) is therefore superfluous, and its superfluity is the sign of a defect, of an imperfection; when we seek to maintain its, then installs a confusion detrimental to the judgment content. The words "relation of a subject to a predicate" design two relations totally different, depending on whether the subject is a predicate or is itself a concept. It would be, therefore, to completely ban the logic of words subject and predicate since they are constantly inciting to confuse the two basically different relation of the sub sumption of an object under a concept of subordinating a concept to another.*[192]

In contrary, Frege's logic is essentially based on symbolic notation. To avoid the ambiguity of this relation, we must renounce to this subject and predicate and substitute the

[191] *Ibid.*, p. 81.
[192] *Ibid.*, pp. 74-75.

couple argument and function. As he affirms "*I think that the replacement of the concept subject and predicate by argument and function, respectively will stand the teste of time*."[193]

In addition to this relation of subject and predicate as constituting the Aristotelian logic which is been replaced by function and argument by Frege, we have the negative judgment and affirmative judgment. In the Aristotelian proposition, there is a distinction between negative and affirmative judgment which are usually classified in what is known as the quality of a proposition. This means if a position is denied or affirmed. Frege thinks that such a distinction is of no interest in logic since there is no law which is been express, this is only an expression of ordinary language. This ordinary language is unprecise in that it does not give us any objective possibility to distinguish between a negative proposition and affirmative proposition. Let us listen to him: "*the same hold for negation […] I find it more appropriate to consider a negation as a brand characteristic of judgeable content*"[194] and not a judgment. Against this this distinction, Frege propose that instead of the distinction of negative and affirmative judgment we should talk of oppose thoughts:

> *The distinction between negative and affirmative thoughts would only be mercrossing things. On the other hand, we can talk of about opposite thoughts. The interest of the notion of opposition is that it is a symmetrical relationship, so that there is no evidence that there is a positive in one case and a negative in another: if A thought has opposed B thought, the thought B is just as well opposed to the thought of A.*

In other word, there is no affirmative or negative thought but only opposite thought since such distinction have linguistic characteristic and not logic; if there exist such a distinction it does not fall within the domain of logic.

5.3.3 On the plan of generalities.

Another difference non-negligible in the comparison of fregean system of logic and that of Aristotle is the expression of generalities. In the Aristotelian system, the generality also known here as the quantity of a proposition, is express through the expression "all", "every" and "no" in contrary, with Frege, the generality is express by the indicative locutions in an indeterminate manner as "something" "that". In fact, according there are three possible expression in general thought "all men are mortals"; "every man is mortal"; "if something is

[193] Gottlob Frege, *Idéographie*, p. 9.
[194] *Ibid.*, 18.

man, something is mortal" and among these expressions the one that seem adequate is the third, meaning the hypothetical proposition.[195] This is because:

> *The expression comporting "all" and "every" are not proper to be employed everywhere generalities appears, because it not possible to sink each law in this mold. In the last mode of expression, new have the form of the hypothetical complex of which, in any case we cannot do without the indicative implements in an indefinite way, "something", "that" and it is this one that the expression of generality is properly found. From this mode of expression, we can easily make the passage to the particular, replacing the indicative phrase undefined by designative phrase in a determined way*.[196]

This means that, for Frege, hypothetical propositions in reason of their possibilities of passing from the universal to particular are the only to express generality or the only judgments in which we can enchant proper names. For example, in the proposition: "***if someone is a man, then someone is mortal***", we can easily substitute the indeterminate indicative phrase by determinate designative phrase as Ebode, Ndzama, Koutchou, Socrate, or with categorical propositions as: "all men are mortals". Let's remind that in the Begriffsschrift, the expression of the generality is symbolized by[197]: 

Arriving at the end of our chapter five, we can recapitulate from this chapter that our focus had been to elaborate Frege philosophical foundation of logic. In the course of our analysis, we came out with the Fregean main symbols and notations which constitute his ideography, the various rules and axioms of his ideography, and how the different inferences are been constructed these axioms. The last is how Frege systems of logic differ with that of the traditional logic of Aristotle in the technical plan, methodological plan and in the generalities. Our chapter six which is the last chapter of our part two will be focused on the Frege's system of logic. Our aim in this chapter is to study in detail the system of the ideography.

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[195] Désiré Ebode, *Systematisation et symbolisation de la logique chez Gottlob Frege : Appropriations et paralogisme d'une revolution*, thesis defended in the university of Yaoundé 1, 2021, p. 177.
[196] Gottlob Frege, *Ecrits posthumes*, Trad. P. de Rouilthan et C. Tiercelin, Hambourg 1999, p. 307.
[197] Désiré Ebode, *Op.cit.*, p. 178.

CHAPTER SIX: FREGE SYSTEM OF LOGIC

In the preceding chapters particularly the three first chapter of part one, we elaborated most pre-fregean systems of logic. Starting with Aristotle, Descartes, Port royal and Kant. Ending with Leibniz and Boole. Now it is time for us to investigate on the system of logic of the person considered as one who revolutionise logic. This investigation will be made into three sub-chapters: the first sub-chapter will be that of though and truth tables where there will be the mathematical square of opposition; the second sub-chapter will be the Vienna circle and the appropriation of Frege logic and the last will be the Tractatus as Frege appropriation of logic.

6.1 THOUGHT AND TRUTH TABLE.

6.1.1. Thought and concept.

Frege

> *Call though of what one can ask if it is true or false. So, he counts among the thoughts what is wrong, just like what is true. He will say: thought is the meaning of any proposition, without asserting that the meaning of any proposition is a thought. The thought itself inaccessible in the sense, has the sensitive coat of the proposition and thus becomes more seizable. We say that the proposition expresses a thought*[198].

This means, that according to Frege, thought is the relation of what is true or false. The possibility of something been true or false is what we call a proposition. Therefore, thought is the meaning of a proposition but it should be noted that not all meaning of a proposition can be consider as a thought. For example, we have imperative propositions, optative's, prayers, exclamations, interrogative proposition that include pronoun. The meaning of a propositions referring here are: complete interrogative propositions, affirmative or communicable propositions of something. According to Frege, our thought is independent of the subject, we are not the owners of our thought like we can have that of representation. This therefore means that: "*thinking it's not producing thought but seize it. What I called thought maintain a close relation with the truth. What I admit for truth, what I judge true regardless of whether I admit*

[198] Gottlob Frege, *Ecrits logiques et philosophiques*, p. 173.

his truth, does not depend on the fact that I think about it. The fact that she is thought does not belong to the true being of thought."[199]

Thoughts are therefore unreal because they are inert against events. seizing a real object quite different of seizing a thought. Once we seize a real object like a cutlass, the later allow a reciprocal transfer of power from one domain to another; it is grasped, to another, while it also undergoes pressure. In revanche, "when we seize a thought, it produces modifications in the only world of the person seizing it, and remain at the heart of her being unchanged. Because the modifications suffered only concern its essential properties. There is missing what we admit in natural event: the effect in return.[200]

Frege considered a concept not as a thought rather as: *"an amount of a determining character that describes the proper of the objects that subsume the concept.*"[201] For example, "rectangle" is the determinant character of the concept "rectangle triangle". A concept is constituted of a property and a character. The author of the ideography, defined these two concepts respectively as: "*all concepts are extension at this extension can be or not be empty. […] The properties of a concept, as having a nonempty extension, having the same extension like the other concept, give itself material to new concepts, insofar as it constitutes them as a determining character.*"[202] Meanwhile, the: "*the character of a concept depict the nature of the objects that a concept could subsume by specifying the properties of such objects, but they do say nothing about the existence of these objects*."[203] To frege, if there must be a relationship between the properties of the objects subsumes by a concept and the character of these concepts, the properties of the concept, are generally, independents of des indeterminant character of the concept that constitute the concept.

If we take into account the above two factors of: properties and character of the concepts proposed by Frege, one will be taken to distinguish three types of concept. The first type commonly called the quality usually refers as Frege put it "*this are concept in the trivial meaning of the term. But not in a precise sense where a concept is a predicate defined on a collection of discreet individuals.*"[204] For example, the word "Red". The second commonly called the unity is when: "*the reflected property and depict by the concept character,*

[199] *Ibid.*, 191.
[200] Désiré Erode, *Op.cit.*, p. 184.
[201] Gottlob Frege, *Fondements de l'arithmétique,* p. 42.
[202] *Ibid.*, pp. 42-43.
[203] *Ibid.*, p. 43.
[204] *Ibid.*, p. 44.

individualize what the subsume, concept."[205] The last is commonly called the concept of superior order which: "*if it is sometime possible to allocate a property that is common to them, these concepts will be gathered under a higher order concept, whose determining character corresponds to the property in question*."[206]

6.1.2. Truth table in Frege's system.

Before any construction of a truth table, it is important that one should analyze the different logical connection on which the rules of the truth table are based. In Frege system of logic, the different symbolism for the construction of its truth table can be seen on his *philosophical writing* and on his *Ideography*.

The negation: to express the negation of a content, we add a to the stroke of the content a negation stroke. For example: Frege symbolize it as ——┐ The negation of a statement is its denial. This means that a negation has the opposite truth value of the statement negated. In other words, it is a truth functional compound statement which is a contradiction to the one stated. Today, modern symbol for negation is: ¬P

P	¬P
T	F

Conjunction: In the case of the conjunction, if we want to put in relation two contents of judgment A and B, the proposition "A and B" are true, if A and B are simultaneously true. Or when the deny of the denial (non (non-A) and B are true or whether the deny of the denial (non (non-B) and A are true. Frege expresses the conjunction as: → Our century uses in substitution of this symbol the notation "and" (.) for most of the English section and the (∧)

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

[205] *Id.*
[206] *Ibid.,* p. 45.

The Disjunctive: a disjunctive is a compound statement made up of two simples statements called disjuncts. This is also known as the alternative or alternant statements since there is just one of the two that have to occurred. The disjunction can be inclusive or exclusive: "*now the words "or" and "either…or" are the used in two way*"[207] Respectively of these words, the inclusive disjunction is expressed by "or" where the proposition A or B are true if one of the proposition is true or is false when both propositions A or B have the same true values which is false-false. Frege represent the disjunctive inclusive a → The mathematical-logic of our century represent the inclusive disjunction as "v"

A	**B**	**A ∨ B**
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

The exclusive disjunction is expressed as "either…or" where either A or B proposition are false when both alternative propositions have the same truth values. This mean that a given exclusive disjunctive proposition to be valid, the rule is that, the truth value is said to be false when both A and B are False or when both true. According to Frege, "the "or" exclusive can be represented as: →. The new formal logic represents the exclusive disjunction as "w".

A	**B**	**A ⩛ B**
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

[207] Gottlob Frege, *Ideography*, p. 18.

The implication: the implication is the conditionality statement made up of two simples propositions the antecedent and the consequence. The antecedent of a true conditional statement provides a necessary condition for the truth of the consequence. The consequence of a true conditional statement provides a necessary consequence of the antecedent. Here, the conditionality is said to be only false when the antecedent is true and consequence false. It is express as "if…then, provided that, a necessary condition" This is represented in Frege ideography as … The modern representation of the implication is (→)

B	**A**	**B →A**
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

The identity: the identity is the relation that hold between two statements when their validity implies each other, meaning having the same truth value. Here, the identity is said to be true when both variables have the same truth values. This is the contrary of the exclusive disjunction in that instead of been false like the exclusive, it is rather true. Another name for the identity is the equivalence which is represented by a "triple' bar (≡) in both Frege and the modern symbolization.

A	**B**	**A ≡ B**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

6.1.3. The square of opposition.

To well understand the square of opposition in Frege system of logic, it is important that we recall the various propositions been elaborated by Aristotle. After this, we will see how these various propositions are been represented by Frege in his ideography and in the last see how these ideographical representations can be read using the modern symbolization. Aristotle brought out four types of categorical proposition classified under two groups quantity and quality.

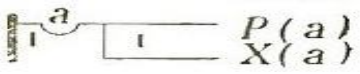
Quantity Quality sign and example

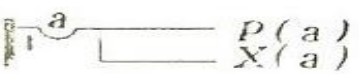
Universal	Affirmative	A e.g. All doctors are researchers.
Universal	Negative	E e.g. No angel are mortals.
Particular	Affirmative	I e.g. Some boys are intelligent.
Particular	Negative	O e.g. Some girls are not givers.

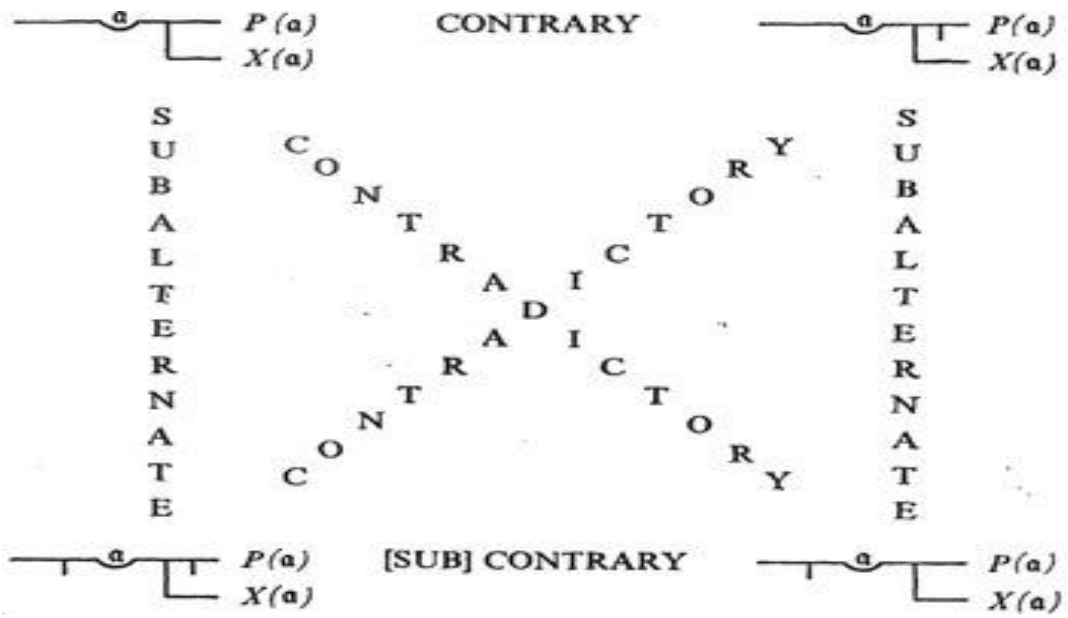
Frege in his ideography uses the generality to express these different propositions in order to build his own squared of opposition. The various propositions can be represented below as:

A: Universal Affirmative 

E: Universal Negative 

I: Particular Affirmative. 

O: Particular negative 



To be able to read this generality found on the square of opposition, we have to note that the Fregean generality is what we called today as the quantification. In the quantification, we have two types: the one expressing the universality and the other expressing the existentiality. Respectively the later can be represented as: **∀x(H(x) → B(x))** and the former as: **∃x(H(x) ∧ B(x)**). It is with the help of these quantifications that we can easily read these different representations. For example,

A: All human are pastors

Symbolization: (∀*x*) (H(*x*) → P(*x*))

Proposition: "for all *x* there exist an x who is a human and this x is a pastor."

"If something has the property H then it also has the property P."

E: no human is a pastor

Symbolization: (∀*x*) (H(*x*) → ¬P(*x*))

Proposition: For all x there exist an x who is a human and this x is not a pastor.

"What has the property H does not have the property P."

I: some human are pastors.

Symbolization: ¬(∀*x*) (H(*x*) → ¬P(*x*)) or (∃x) (H(*x*) ∧ P(*x*)).

Proposition: "there exist an x who is a human and that x is a pastor."

"It is possible for an H to be a P."

O: some human are not doctors.

Symbolization: ¬(∀*x*) (H(*x*) → P(*x*)) or ∃x(H(*x*) ∧ ¬P(*x*)).

Proposition: "there exist an x who is a human and that x is a not a pastor."

"It is not the case that everything which is H is a P."

Given	Contrary	Sub-contrary	Sub-alternation	Contradictory
(∀*x*)(H(*x*)→P(*x*))	(∀*x*)(H(*x*)→¬P(*x*))	Impossible	¬(∀*x*)(H(*x*)→¬P(*x*)) or (∃x)(H(*x*)∧ P(*x*)).	¬(∀*x*)(H(*x*)→P(*x*)) or ∃x(H(*x*)∧ ¬P(*x*))
(∀*x*)(H(*x*)→¬P(*x*))	(∀*x*)(H(*x*)→P(*x*))	Impossible	¬(∀*x*)(H(*x*)→ P(*x*)) or ∃x(H(*x*) ∧ P(*x*))	¬(∀*x*)(H(*x*)→¬P(*x*)) or (∃x)(H(*x*) ∧ P(*x*))
¬(∀*x*)(H(*x*)→¬P(*x*)) or (∃x)(H(*x*) ∧ P(*x*))	Impossible	¬(∀*x*)(H(*x*)→P(*x*)) or (∃x)(H(*x*)∧ ¬P(*x*))	(∀*x*)(H(*x*)→P(*x*))	(∀*x*)(H(*x*)→¬P(*x*))
¬(∀*x*)(H(*x*)→P(*x*)) or ∃x(H(*x*) ∧ ¬P(*x*)).	Impossible	¬(∀*x*)(H(*x*)→¬P(*x*)) or (∃x)(H(*x*) ∧ ¬P(*x*))	(∀*x*)(H(*x*)→¬P(*x*))	(∀*x*)(H(*x*)→P(*x*))

6.2. THE BEGRIFFSSCHRIFT: APPROPRIATION OF THE VIENNA CIRCLE.

6.2.1. The Vienna Circle.

The Vienna circle also called the logical positivism, the logical empiricism, the neo-positivism; inductive logic was an association of scholars born in Germany in the year 1923,

under the impulsion of one of the greatest proponents called Moritz Schlick. The name Vienna circle was given to them because these scholars mostly constituted of philosophers and scientists were meeting every Thursday of the week at Vienna to discourse about science. They had a biblical book which served as a guide and bases on which all their thoughts were focus. This biblical book was the *Tractatus logico-philosophicus* one of the major books published by Ludwig Wittgenstein. The logical positivism can be divided into two; the older and the modern positivism as affirm Popper:

> *The older positivists wished to admit, as scientific or legitimate, only those concepts(notions or ideas) which were, as they put it, "derived from experience"; those concepts, that is, which they believed to be logically reducible to elements of sense-experience, such as sensation (or sense-data), impressions, perceptions visual or auditory memories, and so forth. Modern positivists are apt to see more clearly that science is not a system of concepts but rather a system of statements. Accordingly, they wished to admit, as scientific or legitimate, only those statements which are reducible to elementary (or atomic) statements of experience to judgments of perception or atomic propositions or protocol-sentences or what not. It is clear that the implied criterion of demarcation is identical with the demand for an inductive logic*.[208]

The older positivism was founded at Berlin, by the *Geselischaft fur Empirishche phiolosophie* (society for the empirical philosophy) grouped around fugures likes Hans Reichenbach, C. G. Hempel, R. Von Mises, D. Hilbert, K. Grelling. The modern positivists founded in Vienna was grouped by; M. Schlick, O. Neurath, R. Carnap, V. Kraft, F. Waismann, H. Feigl, L. Wittgenstein.

It should be noted that, the members of the Vienna circle were separated in the year 1936 after the death of Moritz Schlick in the same year killed by one the anti-semit leaner. But the teaching and the focus of this school of thought will not end there.

6.2.2. The Vienna circle and the principle verification.

In order to differentiate what can be called science or not or the demarcation between science from non-science, the logical positivism put in to place a principle on which all what should be called science must be based. This principle is that of verificationnism: "*philosophical doctrine inherited from Ludwig Wittgenstein which only declared admissible only the scientific truth, meaning observable, to the exclusion of all theologico-metaphical*

[208] Karl Popper, *The Logic of scientific discovery (1935)*, London and New York, Routledge, 1992, pp. 11-12.

investigation"[209] This means that for a discipline to be considered as scientific, this must undergo a process of verification which enclose observation. Therefore, "*the meaning of a statement is determined by it […] method of verification*"[210] Meaning that: "*every scientific preposition most be logically reducible to elementary preposition (verification)*[211] In this sense, they apply the concept of sense and references or thought and object been developed by Frege in his philosophical writing. In this approach, any enunciation in logic or science must logically be observe or have a correspondent in nature.

The verification method is based on induction: "*accordingly to a widely view to be opposed in this book, the empirical science can be characterized by the fact that they use inductive method as they are called."*[212] Induction is in fact the procedure by which we can derive universal proposition from particular preposition. By transferring what is observed on certain species case to all the case of these species[213] Meaning that, induction is a process where we pass from singular statements (sometime also called particular statements), such as account of the results of observation or experiments, to universal statements, such as hypotheses or theories[214]. The problem of induction then consists in asking a logical justification of universal statement about reality. [215]

6.2.3. The objective of the Vienna circle.

The logical positivists had a therapeutically objective which was that of quiring science and philosophy from all ambiguity, of language and from nonsensical propositions. Such an objective is also that carry out by Frege and which constitute the aim of his ideography. The logical empiricists by trying out to free science and philosophy from all those propositions that can't be observed or verified, had to eliminate some disciplines from the field of science: metaphysics, ethics, and morals. The elimination of these three disciplines was their main objective since according to them they were speculative and non-sensical. Such an objective

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[209] Roger Mondoue et Philippe Nguemeta, *Vérificationnisme et falsificationnisme, Wittgenstein vainqueur de Popper ?* Yaoundé, L'Harmattan, 2014, p. 37.
[210] Rudolf Carnap, « Le dépassement de la métaphysique par l'analyse logique du langage », in Antonia Soulez, *Manifeste du cercle de vienne et autre écrits*, Paris, PUF, 1985, p. 159.
[211] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, Courrier Cooperation, 1998.
[212] Karl Popper, *The Logic of scientific discovery (1935)*, p. 3.
[213] Moritz schlick, « Le vecu, la connaissance, la metaphysique » in Antonia Soulez, *Manifeste du Cercle de Vienne et autre écrits*, Paris, PUF, 1985, p. 192.
[214] Karl Popper, *The Logic of scientific discovery (1935)*, p. 4.
[215] Moritz Schlick, *Naturwissenchaften **19***, 1931, p. 156.

can be confirming by Popper: "*they are constantly trying to prove that metaphysic by its very nature is nothing but nonsensical twaddle, sophistry and illusion*"[216]

The reason here lied on the fact that: firstly, in the domain of metaphysic (including the philosophy of values and norms) the logical analysis leads to a negative result. The result of the logical analysis is negative because the statements in metaphysic do not obey the necessary and sufficient conditions as to a signification. They violate the logical criterions on which all propositions must undergo to be experimental and precise. These conditions as elaborated by Rudolf Carnap are four of them: **1) the empiric criterion must be known, 2) the statements must be deductible, 3) the truth condition must be established, 4) the verification procedure must be known**. That is why Leclercq could affirm: "*certainly, metaphysical statements have all grammatical appearance of proposition, but a well rigorous logical analysis demonstrate that it is only question of pseudo-propositions which uses meaningless senses or which violate the rules of the logical synthesis*."[217]

The second, is that metaphysics is meaningless. When we say that the statements of metaphysic are meaningless, in a more relaxed use, this is some time taken as a phrase or question that is sterile. In a strict sense, is meaningless a sequence of words which does not constitute a statement within a certain given language. It happens that such a sequence of words appears at first sight to be an utterance; in this case, we call it a half-statement. We therefore support the thesis that the so-called statement of metaphysics is revealed in the light of the logical analysis of the Pseudo-statements. We have two types of pseudo-sciences: either there exists a word been admitted having a signification by errors, or the words really has a signification but form an assembly contrary to the syntax which removes all meaning from them. For example, the words God and principle are terms specifically metaphysical with no signification. This is also the same like "idea, Absolut, unconditional, infinite, being and non-being, emanation, objective spirit, essence, all these are pseudo-statements. [218] As hold Carnap: "*if our thesis that metaphysical statements are pseudo-statements is founded, metaphysic cannot even be expressed in a logically correct language.*"[219] He added by saying that:

[216] Karl Popper, *The logic of scientific discovery*, p. 12.
[217] Bernard Leclercq, *Introduction à la philosophie analytique, la logique comme méthode*, Paris, De Boeck université, 2008, p. 165.
[218] Rudolf Carnap, « Le dépassement de la métaphysique par l'analyse logique du langage », in Antonia Soulez, *Manifeste du Cercle de Vienne et autres écrits*, 162.
[219] *Ibid.*, p. 164.

> *The logical analysis therefore makes its verdict of nonsense against any alleged knowledge that want to have taken from the above or behind the experience. This verdict first reached all speculative metaphysics, any knowledge by pure thought or pure intuition that believes can pass from experience. But the verdict also applies to this metaphysics which, from the experience wants to know by means of particular inference what is outside or behind the experience. In addition, this verdict also applies to any philosophy of values or norms, for any ethics or any esthetics as a normative discipline*.[220]

From the above quotation, the objective of the Vienna circle had been achieved through the verdict render by the analysis of logic. This verdict concern all those disciplines that do not respect the principle of verification or experience. Among these disciplines is at the top all what is said to be above the physical nature which cannot be verified known as metaphysics. The award also concerned ethics, moral and esthetics.

6.3. WITTGENSTEIN APPROPRIATION OF FREGE LOGICISM.

6.3.1. Wittgenstein and the realism of propositions.

Ludwig Wittgenstein of the full name Ludwig Joseph Joh Johann Wittgenstein is a British philosopher from Austrian. Born on the 26 of April 1889 at Vienna and died at on the 29 of April 1951 at the age of 62 years. He was a student of Frege at the university of Jena and Bertrand Russell at Cambridge (1912-1913). Wittgenstein is an author of the most famous book in the history of logic known as *Tractatus logico-philosophicus* in which he tries to follow the part of his master Frege. To understand the book of Wittgenstein, it is necessary to conceive which problem occupies him. There are four problems that touches language: the first is that of psychology, the second is that of epistemology, the third is that of particulars science and the fourth is that of logiques. Among these four, what occupy Wittgenstein is the last:

> *Fourthly, there is the question of the relation that must have a fact with another to be capable of been its symbol. This last is logical question, and it is this that Mr. Wittgenstein is interested. He is interested on the condition of an exact symbolization, that is, of a symbolization in which a statement means something quite defined*.[221]

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[220] This is our translation of : *« l'analyse logiques rend des lors un verdict de non-sens contre toute prétendue connaissance qui veut avoir pris par-delà ou par-derrière l'expérience. Ce verdict atteint d'abord toute metaphysique spéculative, toute prétendue connaissance par pensée pure ou par intuition pure, qui croit pouvoir se passer de l'expérience. De plus ce verdict vaut également pour tous philosophie des valeurs ou des normes, pour toute éthique, ou toute esthétique en tant que discipline normative. », Ibid.*, p. 173.

[221] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trad. fr. Gilles Gaston Granger, Paris, Gallimard, 1993, p. 14.

In this sense, if the symbolization of a statement means something quite defined, this refers to the realism of a proposition. A proposition as defined the author of the Tractatus, is an image of reality. A proposition show it sense. A proposition is a picture of reality. This determines reality to this extend, that one only need to say Yes or No to it to make it agree with reality. This is a description of a fact. As the description of an object describes it by its external properties so a proposition describe reality by the its internal properties. The proposition constructs a world with the help of a logical scaffolding, and therefore one can actually see in the proposition all the logical feature possessed by reality if it is true.[222] The various definition given by Wittgenstein concerning propositions in relation to reality just makes us to apply the theory of Isomorphism.

The theory of Isomorphism is the doctrine which hold the view there is a direct relationship between a statement or proposition with the external world. This means that all the proposition that we advance must be present in nature. That is why Wittgenstein affirms: "*proposition can be true or false only being picture of the reality*"[223] This mean that, the falsity or truth of a proposition can only be verified through the representation that this must have in nature. If the proposition can be verified in reality then this is true but if this cannot be verified in nature then the proposition is false. In this sense, the author is criticizing the traditional logic of Aristotle-scholastics:

> *Most propositions, that have been written about philosophical matters, are not false, but senseless. We cannot, therefore, answer question of this kind at all, but only state their senselessness. Most questions and propositions of the philosophers result from the fact we do not understand the logic of our language. And so, it is not to be wondered at that the deepest problem are really no problem.*[224]

In other words, traditional logic especially that founded by Aristotle do not express reality. It was only full of ambiguity and senseless in content. According to the father of verificationnism all proposition is a picture of reality. The proposition should and is a model of the reality as we think it is. It becomes then imperative that all for a proposition to be considered as been logically reducible this must have a picture in reality or must reflect nature. This is also known as the theory truth-correspondence where by each truth must have a correspondent in

[222] Ludwig Wittgenstein, *Tractatus logico-philosophicus*, London, Kegan Paul, Trench, Trubner & CO. LTD, 1922, pp. 40-41.
[223] *Ibid.*, p. 43.
[224] *Ibid.*, p. 39.

the reality: "*truth is the correspondent to reality or to facts, the agreement or to speak in scholastic terms, the adaequatio rei et intellectus*"[225]

6.3.2. A likeness between Frege and Wittgenstein.

As already mention above in **6.3.1.** Ludwig Wittgenstein has been a student of both Frege and Russell. This is possible that both and especially Frege had a great influence in all most all his thought present in the Tractatus. This can be confirm by himself on the following lines: "*I will only mention that to the greatest works of Frege and the writing of my friend Bertrand Russell I owe in large measure the stimulation of my thought.*"[226] From this quotation, it becomes evident that in the thought of Wittgenstein there is some homology with Frege own thought.

One of the homologies between Frege and Wittgenstein raised on the fight against natural language or as Wittgenstein called it "*everyday language*". According to the first Wittgenstein, there is a need to separate ordinary language from the field of logic and philosophy because this renders language ambiguity and confuse. This is the reason why he affirms:

> *In the language of everyday life, it very often happens that the same word signifies in two different way and therefore belongs to two different symbols or that two words, which signify in different ways, are apparently applied in the same way in the proposition. Thus, the word "is" appears as the copula, as the sign of equality, and as the expression of existence; "to exist" as an intransitive verb like "to go"; "identical" as an adjective; we speak of something but also of the fact of something happening. Thus, there easily arise the most fundamental confusion which the whole of philosophy is full.*[227]

This confusion in which philosophy and particularly logic was full inside, is justified on the fact logic still its origin with Aristotle has always been hitherto followed everyday language and grammar too closely. And this attachment of logic with this language has made it to be polysemous and tautologous. Therefore, ordinary language admits many equivocal terms; ambiguity which should be rejected by scientist.

Against this natural language, Wittgenstein proposes a new form of language which will not more be based on the ambiguity of ordinary language but on mathematic which he called a

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[225] Pascal Engel, *La vérité : réflexions sur quelques truismes*, Paris, Hatier, 1998, p. 13.
[226] Ludwig Wittgenstein, *Tractatus logico-philosophicus*, London, p. 23.
[227] *Ibid.*, pp. 35-36.

symbolic language. This idea of Wittgenstein constitutes another alikeness with that of Frege. The symbolization of language as thought the author of the Tractatus was already an enterprise that was elaborated and implemented by Frege. This symbolism, usually refers to logico-mathematic or universal language is there to render language precise, in such a way that there will not more be synonyms for a given word but each word will denote a specific sign. Such a perspective can be seen in this affirmation of Wittgenstein:

> *In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways. A symbolism, that is to say, which obeys the rules of logical grammar of logical syntax. (the logical symbolism of Frege and Russell is such a language)*[228]

Wittgenstein as his master Frege thinks that the only way in which one can get out of the imprisonment of natural language today, is to implement upon a formula language of arithmetic. Meaning that, we should use mathematical notations and signs to express a given thought or propositions: "we can for example, express what is common to all notations for the truth-functions as follow: by the notations of "~*p*" (not p) and "p ∨ q" (p or q).""[229] Looking at these example, we can observed that a given proposition have been symbolized using the logical connection "not" and "or". The variables "p" and "q" represent the proposition or statement. Literally, the above notation can be writing as: taken p as Paul is married, q as widow. This make, Paul is married or is a widow. Then this is represented as: p ∨ q.

The last but not the least of Frege alikeness with Wittgenstein is concerning the notion of propositions and their contents. Wittgenstein says: "*I conceive the proposition like Frege and Russell as a function of the expression contained in it. The sign is the part of the symbols perceptible by the senses.* "[230] This means that, a proposition is the expression of its constituent and its constituents denote, not only the objects perceptible by the senses, but also express a truth value, either been it false or true. A proposition is therefore not a mixture of words which are abstracts or a skein of concepts devoid of meaning. In contrary, a proposition is an expression of facts, an interconnexion between objects and an expression of truth value. There must be a relation between propositions and objects or with truth value. As affirm Frege: "*we have seen that the denotation if a sentence may always be sought, whenever the reference of its components is involved; and that this is the case when and only when we are inquiring after the*

[228] *Ibid*., p. 36.
[229] *Ibid.*, p. 38.
[230] *Ibid.*, p. 35.

truth value."[231] Meaning that, the constituents of a proposition can be observed in the ability of the proposition been true or false. In this sense to Wittgenstein;

> *Reality must be fixed by Yes or No by the grace of a proposition. A proposition is the description of a fact. The proposition is the description of an object. Meanwhile the description of an object is been done by its external properties, the proposition descripts the reality by its internal properties. The proposition constructs a world with the help of a logical scaffolding, and therefore, we can actually see in a proposition all the logical feature possessed by reality if it is true. One can draw a conclusion from a false proposition.*[232]

6.3.3. Demarcation between Frege and Wittgenstein.

The demarcation between Frege and Wittgenstein is based on two theses; we have the constitution of a proposition and that of truth.

Frege conceived a proposition as the constitution of simple signs or proper names which denote a prices objects in the external world. In this sense:

> *it is clear from the context that by "sign" and "name" I have here understood any designation representing a proper name, which thus has as its reference a definite object (this word taken in the widest range), but not a concept or a relation (…) The designation of a single object can also consist of several words other signs. For brevity, let every such designation be called a proper name*.[233]

This means that a sign and a name are called proper names; a proper name signify an object and this object is it signification; the name is the representation of the object; it has a reference and a sense. To Frege, a proposition expresses an object which is the denotation of a proper name and this proper name is the constitute of sign or symbols and names. "*The regular conjunction between a sign, its sense and it references is of such a kind that to the sign there corresponds a definite sense and to that in turn a definite reference, while to a given reference there does not belong only a single sign."*[234] In clear, to make a short and exact expression possible, let the following phraseology be established. A proper name (sign, word, combination,

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[231] Peter Geach & Max Black, *Translation from the philosophical writing of Gottlob Frege*, Oxford, Basil Blackwell, 1960, p. 63.
[232] Ludwig Wittgenstein, *Tractatus logico-philosophicus*, p. 52.
[233] Peter Geach & Max Black, *Op.cit.*, p. 57.
[234] *Ibid.*, p. 58.

expression) expresses its sense, stands for or designates its reference. By means of a sign, we express its sense and designate its reference.[235]

In contrario, according to Wittgenstein:

> "*the name cannot by analyzed further by any definition. It is a primitive sign. Every defined sign signifies via those signs by which it is defined and the definition show the way. Two signs, on a primitive sign, and one defined by primitive, cannot signify in the same way. Names cannot be taken to pieces by definition (nor any sign which alone and independently has a meaning.*"[236]

This critic of Wittgenstein to the author of the Ideography, makes him to hold that names are simple signs devoid of meanings. They can only have a meaning in a proposition which to the author of the Tractatus this is what expresses a fact, but also is the unique entity of language that has a sense. A proposition become then a constitution of symbols or facts. The author recognize that a name can denote an object, but it is the logical interconnexion that give rise to a fact. Therefore:

> *a proposition is an articulated interconnexion of proper names and conceptual expressions, which fined their signification in itself. That is why, in the same way, the state of things are connections of object (entity, things). Its is part of the essence of a thing to be the constitutive element of a state of things. Certainly, a name is independent of propositions in which it intervenes, since he can intervene in different propositions. But he has a sense because he intervenes on a proposition*.[237]

In a simple way, the proposition is the totality of facts and not things that can exist in the external word. This fact is the reference or designation of the proposition. It is not also a mixture of terms or concept or word but this is articulation of symbols (fact). This fact is only what can express a sense, a class of name cannot.

As enunciated above, another demarcation between the master Frege and the student Wittgenstein is concerning the truth of a proposition. According to the philosopher of the ideography:

> *we are driven into accepting the truth value of a sentence as constituting its reference. By the truth value of a sentence I understand the circumstance that it is true or false. There are no further truth values. Every declarative sentence concerned with the reference of its*

[235] *Ibid.*, p. 61.
[236] Ludwig Wittgenstein, *Tractatus logico-philosophicus*, p. 33.
[237] Bernard Leclercq, *Introduction à la philosophie analytique, la logique comme méthode,* p. 96.

> *words is therefore to be regarded as a proper name, and its reference, if it has one, is either the true or the false. These two objects are recognized, if only implicitly, by everybody who judge something to be true. The designation of the truth values as objects may appear to be an arbitrary fancy or perhaps a mere play upon words, from which no profound consequences could be drawn*.[238]

From this affirmation, it is clear that the truth of a proposition to Frege is it denotation which is that which can be true or false. This denotation in a precise language is the object. Therefore, the truth value of a proposition is not it meaning but its object which constitute it reference or denotation.

This conception of the truth of a proposition will not be accepted by his student. To him:

> *the expression of the agreement and disagreement with the truth-possibilities of the elementary proposition expresses the truth-condition of a proposition. The proposition is the expression of its truth-condition. (Frege has therefore quite rightly put them at the beginning, as explaining the signs of his logical symbolism. Only Frege's explanation of the truth-concept is false: if the true and the false were real objects and the argument in* ***~p,*** *etc., then the sense of* ***~p*** *would be no means be determined by Frege's determination.) […] It is clear that to the complex* ***"F"*** *and* ***"T"*** *no object corresponds; any more than to the horizontal and vertical lines or to brackets. There are no logical objects*.[239]

This means that, for $\boldsymbol{n}$ elementary proposition, there are $\boldsymbol{L_n}$ possible groups of truth conditions. The groups of truth-conditions which belong to the truth-possibilities of a number of elementary propositions can be ordered in a series: among the possible groups of truth-conditions are two extreme cases. In the one case, the proposition is true for all truth-possibilities of the elementary propositions. We say that the truth conditions are tautological. In the second case, the proposition is false for all the truth-possibilities. The truth condition here are self-contradictory.[240] In this case, the proposition has two truth conditions true and false and this refers to two logical class, tautology for all true proposition and self-contradictory for false proposition. In addition to this, Wittgenstein thinks that:

> *all propositions are results of truth-operation the elementary propositions. The truth-operation is the way in which a truth-function arises from elementary propositions. According to the nature of truth-operations, in the same way as out of elementary propositions arise their truth-function, from truth-function arise a new one. Every-*

[238] Frege Gottlob, *Écrits logiques et philosophiques*, p. 110.
[239] Ludwig Wittgenstein, *Tractatus logico-philosophicus*, pp. 51-52.
[240] *Ibid.*, pp. 52-53.

> *operation creates from truth-functions to elementary propositions another truth-function of elementary propositions, i.e., a proposition. The result of every truth-operation on the results of truth-operations on elementary propositions is also the result of one truth operation on elementary propositions*[241].

Meaning that, the truth functions of elementary propositions, are results of the operations which have the elementary propositions as bases. These truth-functions are not material function or function-object as was been defended by the author of the ideography. This refers to facts which are the totality of what exist in the world.

Arriving at the end of our second part entitle: the fregean revolution of logic, it was question here for us to expose in the ideography of the author as a solution to the limits been face by ordinary language and traditional logic founded by Aristotle. For a well and good exposition, it was important for us to analyze the various weaknesses and critics been done by Frege concerning formal logic. Among the critics done by Frege, we found the rejection of terms in a proposition (subject and predicate), the rejection of on the types of judgment, the fight against natural language and the rejection of psychology from the field of logic. After this, we then expose properly the Conceptography of our author. It is important to note that, his Begriffsschrift is divided into three main parts: the first is concern with the explanation of the main symbols been us in to construct the ideography like negation, generality, conditionality. This is fallow by some rules and axioms which constitute the second part of the book. The rules are of several which has been expose here and the axiom are of nine (9) of them: three for conditionality, three for negation, two for identity and one for generality. The third part is the continuation of the second part based on the generality. Even though this part was deserved to the investigation of the ideography, it was important to also mention here some philosophers who influenced our author like Aristotle and to see the demarcation been done by Frege. We also mention those been influence by him like the great school of the Vienna circle without forgetting his student Wittgenstein. This been done, it remains now for us to investigate on the limits of Frege ideography and the philosophical interest this may have today. This will constitute our part three entitle "critical analyses and perspective of the fregean revolution of logic".

[241] *Ibid.*, pp. 61-63.

PART THREE: CRITICAL ANALYSIS AND PERSPECTIVE OF THE FREGEAN REVOLUTION OF LOGIC.

If it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language often almost unavoidable arise concerning the relations between concepts and by freeing thought from that with which only the means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed can become a useful tool for the philosophers.[242]

[242] Frege Gottlob, *Idéographie*, p. 8.

The last part of our work intitled the critical analysis and the perspective of the Fregean revolution of logic, is to help us investigate the limits been encounter by the ideography of our author and the philosophical interests that can be degenerated from his thought. In this case, this part has been divided into three chapters. The first chapter which is the chapter seven of our work will be to expose the weaknesses that the concept-script face to be well establish. The remaining two chapter will be to expose the philosophical interest: the chapter eight particularly will be the challenges of the revolution of Frege in logic. In other words, this will be to give the impacts that the Fregean revolution may have in the epistemological, social and linguistic domains. The last chapter which is the chapter nine will present the relation between the Fregean revolution of logic and Africa today. This means that, it will be the occasion for us to see how Frege thought (ideography) can be useful in the African continent in present days. How can his Conceptography be important to the development of the black people. These three chapters as already mention will be the so close of our work in this part.

CHAPTER SEVEN: THE RELEVANCE CRISIS OF THE FREGEAN LOGICISM.

Human beings are fallible and this show their imperfection. If man is imperfect, this means that he can't do something that is perfect. The ideography which is written by a human being can't be consider as been perfect since this has some limits or weaknesses. It is this limits that will constitute the investigation of our chapter seven. This chapter therefore consist of analyzing the different crises been face by the theses of our author. This chapter will be divided into three sub-chapter. The first will show the limits of Frege in founding a universal language due to the present of multi-language, mutation of language and undecidable propositions. This will be followed in the next sub-chapter by Frege as anti-evolutionism as an establishment of a precise and fixed language.

7.1. THE HINDERANCE IN FOUNDING A FORMAL LANGUAGE.

7.1.1. The presence of multilingual.

Language in the broad sense; is any communication by signs. In the strict sense, or specifically human: the ability to speak or all human language. This then means that language is perceived as the plurality of languages that exists in our world which constitute what we call multilingual. The multilingual is therefore, the presence of the plurality of languages in our world. We traditionally distinguish five main continents: America, Asia, Africa, Europe and Oceanic. These five continents have each a number of countries. The number of countries today are around two hundred. These countries each have at least one or more than one official languages. The numbers of languages present in the world is estimated to be more than seven hundred (700). Among which we have: English, French, Germany, Latin, Spanish and Greek. The presents of these several languages in the world become a hinderance to the formulation of an artificial language and universal language as imagined and thought Frege: "*I did not only want to create a calculus ratiocinator but a lingua characterlike in the sense of Leibniz*".[243]

The plurality of language is a limit to the formulation of a formal language in that, the world is not made up of a language but languages. So, to formalize these languages becomes a problematic and even impossible since we must take into consideration all these languages. The

[243] Gottlob Frege, *Ecrit logiques et philosophiques*, p. 71.

consideration been taken is not forgetting that a word in each language may signify several things in other language and that we may have the same word in different language but with different meaning. For example, the word "land" in Germany is a country, in English is a territory in French and is a common name. And this is what Frege want to prevent or avoid in his logic. To him, each word must have a specific meaning to all independent of the language. But such a conception hinders a problem in its applicability to each language. This could be possible if the world had only one language as it was in the ancient day with the tower of Babel where the people where using the same language for communication.

Even if one succeeded in implementing this in a given language, the other will receive this as a translation and once there is a translation then we betray the original language since this will not be translated with the same originality like the first. This can be seen with the symbols been used in logical connectors. We have for example, the sign **"und"** in Germany meaning "and" which to Frege is given as **"→"** in French the same word is **"et"** is represented as **"∧"** and the same word in English "**and**" is represented **"."**. From this, we cannot talk of artificial and a universal language but of artificial and universal languages since each language will give and have its own way of formalizing its own word. This bring us back to the ordinary language.

7.1.2. The mutations of language.

The hindrance of formalizing a language is not only as a result to the plurality of language. This can also be seen with the mutation of language. By mutation of language in the general term is the dynamic change of something either in a short or long run. In the linguistic domain, mutation is the change in vowel sound caused by a sound in the following syllable. Frege by establishing his theory of sense and references where: "*the regular connexion between a sign, its sense and its reference is of such a kind that to the sign corresponds a definite sense and to that in turn a definite reference*"[244] This means that, to each sense or thought there is a definite reference or object. A word thus has only one meaning and this meaning refers to a specific object. This is to fight against the synonyms of a words which usually denote many objects.

The problem of such a relation is that, this does not take into consideration neither the dynamic of language nor the contribution in the determination of the sense of a proposition.

[244] *Ibid.*, p. 101.

Since the evolution of man, the use of communication has always been changed according to time and space. According to time, there are some words that where been having a certain signification which today these does not more have the same meaning. For example, the word "silly" which means today somebody who acts in a draft manner had a different meaning before as "something far more serious". The word "flux" which it original meaning was "diarrhea or dysentery" today means "Constance change".

The formalization of language becomes impossible to be established under such condition since if one word has a definite reference or denotation and that this word has to change its meaning, we will be up large to look in this case for a new reference so as to give to this word a definite or specific denotation. In addition, to this each time there are new words that are been created in the domain of communication and which enter into the dictionary of existing words. For such word, we must also look for a given notation that can represent its symbolization.

7.1.3. The present of undecidable propositions.

Frege hold the view that all propositions must have a thought and a denotation, is excluding all those propositions that do not have a reference or denotation as been senseless. To him therefore, we must be capable of representing all the propositions been enunciated so as to apply the theory of sense and reference. The reference of the sense of a proposition is either given through object or truth value. By holding such a view that all propositions must be represented, the author of the Ideography seems to forget that not everything does can be represented or formalize using symbols. This is what we usually call the unrepresentative or undecidable propositions. The undecidable propositions appear in this sense as those propositions that exist but which cannot be express or represented through the use of symbols and notations. Then, the unconceivable and undecidable fall under the unrepresentable. Among what is unrepresentable, meaning unconceivable, we have propositions concerning the emotional feeling, proverbs, songs and myths. For example, how can we represent the love one has for another person? Either his mother, wife, husband or child. How can we represent the proverb "a hungry man is an angry man"? How can we symbolize the song of a bird?

It is in this light that one of the German philosopher and mathematician Kurt Gödel[245] through his book *On formally undecidable propositions of principia mathematica and related system* demonstrate that: "*every system of arithmetic contains arithmetical propositions, by which is meant propositions concerned solely with relations between whole numbers, which can neither be proved nor be disproved within the system.*"[246] This means that, the Gödel's theorem, as a simple corollary of propositions is frequently called, prove that there are arithmetical propositions which are undecidable (i.e. neither provable nor disprovable) within the system, and the proof proceed by actually specifying such a proposition, namely the proposition g expressed by the formula to which "17 Gen r". g is an arithmetical proposition; but the proposition that g is undecidable within the system is not an arithmetical proposition, since it is concerned with provability within an arithmetical system, and this is a meta-arithmetical and not an arithmetical notion. Gödel's theorem is thus a result which belong not to mathematics but meta-mathematics, the name given by Hilbert to the study of rigorous proof in mathematics and symbolic logic.[247]

From this point of view, the mathematical logic been put forward by Frege becomes limited since the nine (9) axioms and the difference rule of inference in which his ideography is based, most of this can't be demonstrated nor refutable. In this case, they are considered as

7.2. FREGE, AN ANTI-EVOLUTIONIST.

7.2.1. Implementation of mathematical symbols in language

Frege criticize ordinary language as been unprecise and polysemous. The defects that we have pointed out have their origin in a certain instability and mutability of the language, which are moreover the condition of its faculty of evolution and of its multiple resources. For this purpose, language can be compared to the hand which, despite its ability to perform extremely diverse tasks, is not enough for us.[248] Frege thinks that today if our language is imperfect, is because this is always changing from time to time. In this case, we make for ourselves artificial hands, tools designed for special purposes and which accomplish the work with a more precision of which the hand was incapable. These artificial hands are what we

[245] Kurt Gödel, of the full name Kurt Friedrich Gödel is a German philosopher and mathematician of the XXth century. Born on April 28, 1906 and died on January 14, 1978, he is mostly known for his theory of Gödel incompleteness and completeness theorem.
[246] Kurt Gödel, *On formally undecidable propositions of principia mathematica and related system,* Trad. En. B. Meltzer and R. B. Braithwaite, New York, Dover Publication, 1992, Introduction, p. 1.
[247] *Ibid.,* pp. 1-2.
[248] Gottlob Frege, *Ecrit logiques et philosophiques,* p. 66.

called sign or symbols which are purified from all ambiguity and which do not take into consideration the context. The reason for Frege implementing signs or symbols into language is because:

> *The visible signs, and in particular the figures, have another nature. The figures are generally well delimitated and clearly differentiated. This precision of the written sign will result in given a clear relief to what is designated. […] and another advantage of the written sign comes from its great duration and its immutability*[249]

Meaning that, the author of the *Ideography* wants to complete the language with mathematical formula by means of signs or symbols.

From this quotation, it can be seen that our author is against the evolution of language. This is why he proposes mathematic signs so as for language to be fixed as he called it immutable. We can in this case qualify him as an anti-evolutionist. To him therefore, language must remain static in such a way that the same sign been used to denote a particular word must remain forever in the formula language. The mutation of language is in this way stable: the meaning of a given word remains precise by denoting the same thing as time passes. For example, the sign for **"and"**, "**or"** must remain the same as "**∧"**, "**v**" respectively independent of the time, area or context in which the word is been use or employ.

7.2.2. The rejection of the evolution of logic.

One of the theories which was dominating the mid-nineteen centuries in science was the theory of evolution hold by the English naturalist Charles Darwin. His theory of evolution is known as the Darwinism. The Darwinism is a doctrine of biological evolution which holds that all species of organisms arise and developed as a result of natural selection, inherited variation that increase the individual's ability to compete, survive and reproduce. This theory was now been used to show the view that the universe and life in all its manifestations and nature in all their aspects is the product of development as affirm Frege:

> *In this time when the theory of evolution is marching triumphantly through the sciences and the method of interpreting everything historically threatens to exceed its proper bounds, we must be prepared to face some strange and disconcerting questions. If man, like other living creatures, has undergone a continuous process of evolution, have the laws of his thinking always been valid and will they always retain*

[249] *Ibid.,* p. 67.

> *their validity? Will an inference that is valid now still be valid after thousands of years and was it already valid thousands of years ago?*[250]

The author answered to this question by a positive by saying that the law of thought(logic) has always been unchanged. In the sense in which we speak of logical laws, it is, strictly speaking, impossible for laws to change at all. For such a law, expressed in full, must include mention of all relevant conditions, in which case it will hold independent of time and space. The law of inertia, for instance, claims to be valid for all time and region of space. In this case, Frege affirms: "*if by the laws of thought we understand the law of logic, it is easy to see the absurdity of a condition relating, say, to the phosphorus content of our brain or to something else in human beings which is subject to change.*"[251] This means that, 2 times 2 is 4 is true and will continue to be so even if, as a result of Darwinian evolution, human beings were to come to assert that 2 times 4 is 5. Every truth or law is eternal independent of being thought by any one and of the psychological make-up of anyone thinking it.

From the above argument, it can be deduced that there is no relationship between the Darwinism theory of evolution and logic since there is no direct consequence that follow from it. Meaning that, the Darwin theory is itself exceedingly interesting, but it is not the kind of fact from which philosophical consequence follow. This is the reason why Wittgenstein following the path of Frege affirms:

> *Philosophy is not one of the natural sciences (the word philosophy must mean something which stands above or below, but not beside the natural sciences.) The object of philosophy is the logical clarification of thoughts. Philosophy is not a theory but an activity. Then, the Darwinian theory has no more to do with philosophy than has any other hypothesis of natural science.*[252]

The law of thought known to be logic has never followed any evolution, will not follow any evolution and will never follow any one in the next thousands of years. Such argument makes Frege to be considered not only the enemy of progress but also a real anti-evolutionism concerning the change or development of logic.

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[250] Gottlob Frege, *Posthumous writing*, Trans. En. Peter long & Roger White, Oxford, Basil Blackwell, 1979, p. 4.
[251] *Ibid.*, p. 5.
[252] Ludwig Wittgenstein *Tractatus Logico-philosophicus*, London, Kegan Paul, Trench, Trubner & CO. LTD, 1922, pp. 44-45.

7.3. THE DIFFICULTY IN MASTERING THE GRAPHICS

7.3.1. The linguistic complementarity.

The linguistic complementarity was a principle put forward by the philosopher Lofgren Lars, which hold that: "*in no language (i.e. a system for generating expressions with a specific meaning) can be process of interpretation of the expression be completely described within the language itself.*"[253] In other words, the procedure of determining the meaning of expression must involve entities from outside the language, what we have called the context. The context becomes in this case the circumstance that form the setting for an event, statement, or idea, and in terms of which it can be fully understood. Meaning that, this is the time or place in which the word is been use to describe it full meaning. The context of a term must be taken into consideration when trying to understand the meaning of that term. The reason lies on the fact that, the terms of a language are finite and changeless, where as their possible interpretations are infinite and changing. When we say that the terms of a language are finite and changeless, we mean that in a sentence, we have a given number of terms that constitute that sentence but the fact that this number of terms are finites those not infer that the sentence been writing must means only one thing as though Frege. That is why we say the possible interpretations are infinite and change. In this case, to know the finite meaning of the expression, we must take into consideration the circumstance at which this expression has been said or used.

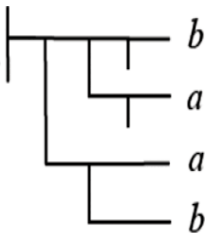
For instance, in a text, to understand the meaning of a particular phrase or piece of text it is important to read what comes immediately before and after. Example, the affirmation "**anything goes"**[254] This is a phrase been asserted by Paul Feyerabend. When we grasp the phrase from its appearance, we can say the author want to say all is good. This may not be false but to well understand the meaning of this sentence, it will be important to search for what have been said before in order to arrive at this assertion. This is why we could say with the second Wittgenstein: "*in this sort of predicament, always ask yourself: how did learn the meaning of this word? From what sort of examples? In what language-games? Then it will be easier for you to see that the word must have a family of meaning*."[255] So, "*the meaning of a word is its*

[253] Francis Heylighen, "Advantages and limitation of formal expression", in *Foundation of science 4*, Center Leo Apostel, march 1999, p. 8.
[254] Paul Feyerabend, *Contre la Méthode*, trad. fr. Baudouin Jurdant & Agnès Schluberger, London and New York, New Left Book, 1975, p. 323.
[255] Ludwig Wittgenstein, *Philosophical Writing*, trans. en. G. E. M. Anscombe, P. M. S. Hacker & Jaochim Schulte, United State, Wiley Blackwell, 2009, p. 32.

use in the language"[256] With this, the elimination of the context in a proposition in formalize language becomes impossible.

7.3.2. Ideography as a challenge for human cognition.

Acquiring knowledge for human being is a step by step process that takes time to be assimilated. The human brain is divided into three main parts and each of these has a specific function: the **frontal lobe** responsible for emotions, the **Broca's area** with function linked to speech production, and the **motor cortex** responsible to generate signals to direct the movement of the body. Our main concern is for the Broca's area which is responsible to produce communication. The ideography is a sort of communication mostly consider as written communication. This mostly uses mathematical signs and some graphics which help to elaborate all the language. For example; for the mathematical sign we can use ∧, which means "and" → meaning "imply", ≡ which refers to "identity". Such signs are easier to be comprehended and memorize by our cognition. The difficulty appears when these signs or graphics become complex. This complex of the scriptures causes a serious problem for our mind to memories them. Let look at these examples,



These two graphics above are those propose by Frege[257] which have each other their meaning. The first can be represent in the logical form as: ⊦**(b→a) → (¬a→¬b).** Literally, this can be read as "if **a** is the necessary consequence of **b,** this implies that **¬b** is the necessary consequence of **¬a.** The second graphic is a universal quantification which can be represent logically as **(∀*x*) (H(*x*)→P(*x*)).** In the case where our language has to be made by several of such graphic to be understood and master for communication, we see that this will be very difficult for our brain to master all these graphics. This is because: the complex scriptures require more time, attention and cognitive processing to be produced and understood. The absence of context forces the language user to code the necessary presumptions within the

[256] *Id.*
[257] The two types of graphics above have been taken from Gottlob Frege, in his *Begriffsschrift*, p. 30 for the first and p. 27 for the second.

message. The resulting syntactic mode of expression involve a higher use of noun that required more lexical searching because of their relatively infrequent use." This case,

> *The human mind cannot memorize large numbers of pairings between meanings and visual symbols. A self-sufficient code can be built on the basis of a few conventions, as long as it remains specialized and follows relatively strict rules of composition. But a more generalist code, to be usable without oral gloss, would require users to learn an excessive number of conventions. If human only have a large memory storing for codes and symbols when symbols are made of phonemes, this would solve the puzzle of the ideography*.[258]

In other words, a language composed of a general symbol will be very difficult for people to be understood and learn. To do this, we must specialize in to the science that will help us to read them. This is contrary to ordinary language where you do not need to specialize, to know or understand the language of a given people since this can easily be accessible to each and every one. The possibility of forgetting such a language is of low percentage since it is very easy to retain a spoken language than a writing language which you will not frequently be using. Frege language is a writing language that those not have any connection with the spoken language as been develop on his philosophical writing.

7.3.3. The inflexibility of the ideography.

By flexibility here, we understand the capacity of an object or subject to be capable of moving, changing, of shifting after a given phenomenon appears. So, the inflexibility of the ideography resides on the bases that this is static or rigid. By inventing the ideography, Frege aim at solving the problem of the evolution of language which render it to be ambiguous. The ideography to frege at as function to treat the misunderstanding and errors of the ordinary language:

> "*The defects that we have pointed out have their origin in a certain instability and mutability of the language, which are moreover the condition of its faculty of evolution and of its multiple resources. For this purpose, language can be compared to the hand which, despite its ability to perform extremely diverse tasks, is not enough for us. we make for ourselves artificial hands, tools designed for special purposes and which accomplish the work with a more precision of which the hand was incapable. These artificial hands are what we called sign or*

[258] Olivier Morin, *The Puzzle of ideography*, https//www.shh.mpg. de//94549/themintgroupe, consulted on the 30th of May 2023, at 02 a. m.

> *symbols which are purified from all ambiguity and which do not take into consideration the context*."[259]

By creating such a language, Frege forgot that a language can't be static in all the given context. It is important a given language or expression should be said or adapt to a given context or domain in which we are found. Meaning that, each domain or context necessitate a change in the way of communicating. That is why our language is flexible according to where we are. For instant, the language been use in communicating in geography is different from that been used by mathematician, which itself is different from that been used in philosophy, economics or historian. Also, the speech which I hold in front of the leaners are different from that been hold in front of the villager even if the theme is the same. This is the reason why the student of Frege could write in his *Philosophical investigations*: "*the word ''language games must bring out here that talking of language is part of an activity or a form of life*".[260] Meaning that, the question of language is now gnoseological. This means, language depend on the socio-cultural area in which this is been used. For each culture or a society, correspond a given language and conception of things which is different from the conception of the others. The ideography does not accommodate itself to instance of the different phenomena.

To Francis Heylighn and Jean-Marc Deweale, the concurrent disadvantage of invariance over contexts is that formal speech is more static or rigid, and will less easily accommodate to phenomena that demand on expressions with a meaning different from the ones found in the dictionaries. Informal speech, by definition, is flexible: meanings shift when the context changes. This is particular useful when phenomena are to be described for which no expression is available in the language as get. By using context-dependent expressions like "it" or "that thing there", it is possible to refer to the most unusual phenomena.[261]

To conclude this our chapter seven, we can say that our main preoccupation has been that of elaborating the different crises hinders by the Fregean logical revolution to be establish. The first difficulty encountered by Frege was the hinderance in founding a formula language

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[259] This is our translation, « *les défauts que nous avons signalés ont leur origine dans une certaines instabilité et mutabilité du langage, qui sont par ailleurs la condition de sa faculté d'évaluation et de ses ressources multiples. Le langage peut à cet effet être comparé à la main qui malgré sa capacité à remplir des taches extrêmement diverses, ne nous suffit pas. Nous nous faisons des mains artificielles, des outils conçus pour des but spéciaux et qui accomplissent le travail avec une précision dont la main n'était pas capable* » in Gottlob Frege *Ecrit logiques et philosophiques,* p. 66.

[260] This is our translation of "*Le mot « jeux de langage » doit faire ressortir ici que le parler du langage fait partie d'une activité ou d'une forme de vie » in* Ludwig Wittgenstein, *Investigations philosophiques*, p.125.

[261] Francis Heylighn & Jean-Marc Deweale, "Formality of language: definition, measurement, and behavioral determinants" in *internal report, center "Leo Apostel" Free university of Brussels*, Heylighn & Deweale, 1999, pp. 9-10.

due to the present of multi-languages which are about seven thousand in the world, the mutation of this languages according to time and space and the present of undecidable proposition which cannot be proven nor represented. The second is that Frege is considered as an anti-evolutionist which can be seen with the implementation if fixe symbols in language making the language to be fixe without any mutation meanwhile language has to be dynamic. We have the rejection of any evolutionism in logic. To him, the laws of logic has always been the same. The third is that of the mastering of this ideographic symbols which takes time to be master and to retain all the laws going with them. This means the notations and signs present in the ideography necessitate one to have an opening mind and to like mathematics.

CHAPTER EIGHT: THE CHALLENGES OF THE REVOLUTION OF LOGIC IN FREGE.

After given the limits and critics encounter by the ideography of Frege as the formula language, modeled upon that of arithmetic, for pure thought in the above chapter, it is time for us in these two remaining chapters to bring out the challenges that can have the thought of our author. The chapter eight in which we are concern now will be carry on the challenges of the Fregean revolution in the epistemological domain with the social domain and the linguistic domain.

8.1. EPISTEMOLOGICAL CHALLENGES.

8.1.1. Store knowledge in the long run.

Knowledge is facts, information and skills been acquired through experience, reasoning, experimentation, studies or education. Meaning that, this is the theoretical or practical understanding of a subject, or object. This knowledge can be in the form of science, art or technic. Scientifics knowledge are mostly theoretical, artistic knowledges are both theoretical and practical and technics knowledge are mostly practical. Frege language which is that of his ideography, defined as the formula language, modeled upon that of arithmetic, for pure thought allow knowledge to be stored in the long run. Most of our natural languages are said to be dynamic. The mutation of language is due to the fact that this take into consideration the concept of space and time. The impact of such consideration is that, this render information to be stored temporally. Meaning that, knowledge of ordinary language is stored for a short run since these may change after a given time and in a given territory.

Indeed, the language proposes by Frege is that which permit us to stored our information for a durable period of time. This language is what we called the formula language which is a form of language based on mathematical notions or symbols especially that of arithmetic. The use of arithmetic signs in language render it to be durable and immutable:

> "*one of the advantages of written sign comes from its greater duration and its immutability. Through these characters, it is similar to the concept, as it should, and all the more dissimilarly the incessant current of our actual thoughts. Scripture offers the possibility of retaining*

> *several elements simultaneously, and even if the eye can only seize that a small part of the context, we keep a general impression that remain s at our immediate disposition, according to our needs. The relative position of the written signs distributed in the bi-dimensional writing plan can be used to express internal reports, more deriva than the simple procession of uni-dimensional time. […] a writing who want to exploit all the advantages specific to visible signs must be entirely different from all spoken languages.* "[262]

This means that, the duration and an immutability of written sign in language permit us to keep information for future use without any change in its meaning or orthograph. Since the meaning of a formal expression is by definition independent of the time, that meaning remain for future uses. The longer we desire our language expression to remain meaningful, the more formal we should try to make. By making our language formal, that is using scripture, this will permit to guarantee the rigor of the course of thought which cannot be possible with our language since it is not governed by the laws of logic. The scripture or symbols are important for written language due to the fact that the scripture takes precedence over the spoken word (ordinary language). One can regard several times a sequence of thoughts without fear that it is impaired and checked its concluding values as well. The logical rules are then applied from the outside like a barrel, since the simple writing of the words of the spoken language does not offer, by nature, no logical warranty. The fact that we do not fear for the impaired of symbols shows us that the formulisation of language will help to store our information and our language will remain static and unchanged.

8.1.2. Formula language: Precise understanding and translation of knowledge.

One of the fundamental problems in man process of acquiring knowledge is the capacity of understanding and translating what is been thought to him. By understanding, we mean the faculty of thinking the object of sensuous intuition; or the faculty of concepts, judgments and principles. The understanding is the source of concepts, categories and principle by means of which the manifold of sense is brought into unity of apperception. Meaning, this is the faculty that we can interpret things from the association of sense and reasoning.

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[262] Frege Gottlob, this is our translation of « *un autre avantage du signe écrit lui vient de sa plus grande durée et de son immutabilité. Par ces caractères, il est semblable au concept, comme il se doit, et d'autant plus dissemblable du courant incessant de nos pensées effectives. L'écriture offre la possibilité de retenir présents plusieurs éléments simultanément, et même si l'œil ne peut saisir à chaque regard qu'une petite partie du contexte, nous gardons une impression générale qui demeura à notre disposition immédiate, selon nos besoins. La position relative des signes écrits distribué dans le plan d'écriture bidimensionnel peut servir à exprimer des rapport internes, de manière plus déliée que le permettent les simple précession et succession du temps unidimensionnel […] une écriture qui veut exploiter tous les avantages propres aux signes visibles doit être entièrement différente de tous les langages parlés* » in *Écrits logiques et philosophiques*, p. 67.

Translation refers her to the capacity of someone to reproduce either through oral or written what he has acquired. The aspect of reproducing a thought is our capacity of understanding the thought. It appears that our knowledge is facing a greater problem in its understanding and translation. This problem is that of the ambiguity of natural language and the overabundance of words having a blurred meaning. Frege in this sense affirm:

> *"the language is defective when it comes to prevent the errors of thought (knowledge). it's does not satisfy the essential condition, that of univocity. The most dangerous cases are those where the meanings of words differ, where the variation are light although not equivalent.*" [263]

For example, the word "is" can signify at the same time a copular, the sign of identity, the expression of existence, an intransitive verb, as an adjective. We see that such a word makes knowledge to be misunderstood and this will have a difficulty in translating the expression that may come from it. Also, it is not evident that one reproduced exactly an information that has been translated to him orally to be given to another person. There will be a modification in the retranslation of this information. In this case, a formula language becomes the solution that can help us to remedy such a misunderstanding and misinterpretation. Through the use of mathematical symbols in language, we are sure that our understanding of the various knowledge been acquire will be high and this can be reproduced with precision without taking into consideration any characteristic of the physical, social and mental situation that can influence this understanding and interpretation of an expression. Thus,

> "*we defined formality as avoidance of ambiguity in order to minimize the chance of misunderstanding. This means, first of all that formality will be highest in those situations where accurate understanding is essential, such as contracts, laws, or international treaties. Second, formality will be higher when correct interpretation is more difficult to achieve. One way to secure accurate understanding is corrective feedback: if the listener can signal to the speaker when he or she doesn't understand, so that the speaker can reformulate the phrase, the speaker will need to worry less about ambiguous expression.*"[264]

From the above quotation, it is clear that the most fundamental mission of the formality what we called formula language in Frege is the unambiguous the ordinary language by

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[263] Frege Gottlob, this is our translation of « *le langage se révèle défectueux lorsqu'il s'agit de prévenir les fautes de pensée. Il ne satisfait pas à la condition ici primordiale, celle d'univocité. Les cas les plus dangereux sont ceux où les significations des mots diffèrent très peu, où les variations sont légères bien que non équivalentes.* », in *Écrits logiques et philosophiques*, p. 64.

[264] Francis Heylighn & Jean-Marc Deweale, "Formality of language: definition, measurement, and behavioral determinants" in *internal report, center,* pp. 25-26.

reducing the level of misleading expressions and the level of misinterpretation of our knowledge. This unambiguity of language has been creating confusion in the field of philosophy and science. Today, we think that formula language developed by Frege will be an advantage for us to evacuate such a language due to its precise and rigorous way in which his formality has been elaborated to be used.

8.1.3. The invention of quantifiers[265] and variables

Etymologically, this come from the Latin word "*quantificare*" this means quantifier. In traditional logic, this designates all the synamental markers by which is specified the quantity or the extension of a concept, proposition, subject or a predicate. For example, All, Some, At least one, little. All students are wise. In modern logic, this is the operator of the calculation of predicates which specified in a proposition the extension or the logical quantity (partial or total) of the variables and allow the exchange to the propositional function a value of truth. The quantization can wear the variable of individuals (logic or calculating first arrange) and on the predicate variables (logic or second order calculation of the predicates). The operator formally translates the various quantifiers of the natural language.[266] The quantifier was invented by Frege as a solution to well formalize the various quantifier of natural language. There exist two types of quantifiers:

1) The universal quantifier.

The universal quantifier designates in a proposition the complete extension of a predicate in regard to its subject. With its propositional argument, the universal quantifier hold that a proposition is true for all the variables identified. This is represented by the invers of the A letter. The symbol is given by **∀** and this is been red as "for all". After this symbol which is the quantifier, we have another letter beside which represent the variable. The variable is usually given as **x**. The final symbol becomes: **∀x.** this is been red as "for all x there is". The implication sign (**→**) is what links the various predicates in the universal quantifier. For example, all learner are searchers. Given L as learners and S as searchers, we can simplify the following as:

$$\forall x\,(Lx \rightarrow Sx)$$

[265] The device of quantification is carried out in sections 11 and 12 of Begriffsschrift, where Frege sets up a way in which the complexity of a proposition is accounted for terms of variables and quantifiers. This has already been developed in our work in 5.1.3. pp. 75-76.

[266] Steeven Chapados, *Dictionnaire philosophiques et historique de la logique*, Laval, PUL, 2017pp. 340-341.

The given symbolization can be read literally as; for all x, if x is a learner then x is a searcher. Or simply as: for all x, if L of x, then S of x.

Another example of a universal quantifier is, All Ph. D are either researchers or lecturers. Given x as Ph. D, R as researchers and L as lecturers, we obtain the following:

$$\forall x\,(Rx \rightarrow Lx).$$

This is been red as: for all x, either x is researcher or x is a lecturer. Or for all x, R of x, or L of x. It is important to note that the logical connectors of "and" and "or" to Frege are all implemented in the implication sign. That is why our logical connector in the symbolization still remain that of implication. But today, with the modern representation this has changed. Our representation can be given as:

$$\forall x\,(Rx \vee Lx).$$

2) The existential quantifier.

Before analyzing this existential quantification theory, it is important to keep in mind that, "*Frege's logical symbolism contain only one quantifier, the universal quantifier, since, Frege's logic being classical, the existential quantifier is expressible in terms of the universal one together with negation.*"[267]

The existential quantifier designates in a proposition the partial extension of a predicate in regard to its subject. With its propositional argument, the existential quantifier hold that a proposition is true at least for an individual variable. This is represented by the invers of the E letter. The symbol is given by ∃ and this is been red as "there exist". After this symbol which is the quantifier, we have another letter beside which represent the variable. The variable is usually given as **x**. The final symbol becomes: **∃x**[268]**.** This is been red as "there exist". For example, some students are hard working. Taken S for students and H for had working, we have:

$\neg(\forall x)\,(S(x) \rightarrow \neg H(x))$ for Frege = $(\exists x)\,(S(x) \wedge H(x))$ for modern.

[267]Michael Dummett, *Frege philosophy of language*, New York, Happer and Row publishing, 1973, p. 512.

[268] It should be noted that the symbol for existential quantification was introduced. Frege himself does not introduce a special quantification to represent the word "some". For it does not seem indispensable to have a special notation for the existential quantifier, in so far as it had long been assumed by some logician that a sentence such as "some swans are blacks" is equivalent to "not all swans are not black. Frege makes uses of this relationship between "some" and "not all…are…. not" in order to express an existential proposition.

In Frege's symbolization this will be red as: not all S of x are not H of x. Or there exist an x, where S of x is H of x.

In frege quantifier we can have two or more quantifiers and variables in a given proposition which is not the case with the Aristotelian traditional quantity and Boole:

> *The essence of Frege's advance beyond Boole was the appearance, in 1879 of truth function connective within the scope of quantifier, along with quantifiers within the scope of truth functional connective. But simultaneously there occurred a yet greater advance: the appearance of quantifier within the scope quantifier.*[269]

Example: "everybody loves someone"

This can be represented as: **(∀x) (∃y) L x y**

8.2. ON THE LINGUISTIC DOMAIN.

8.2.1. Frege logicism: the universalization of language.

The expression of logicism course pejoratively, in the linguistic register at the end of the nineteenth century as the analysis of natural languages by means of concepts borrowed from logic especially mathematical logic.[270]Meaning that, logicism is the formula language where mathematical symbols or notation are used in our everyday language so as to render it more precise and rigorous. Such an idea was thought by Leibniz but was really implemented and developed by the mathematician Gottlob Frege in 1879 and 1884. Frege analyze that our ordinary language was suffering from ambiguity which make our language to be defective in oral and even written. For this reason, he invented the ideography which he defined as a formula language, modeled upon that of arithmetic, for pure thought. That is why he affirms "*in attempting to comply with this requirement in the strictest possible way I found the inadequacy of language to be an obstacle… this deficiency led me to the idea of the present ideography.*"[271] The ideography becomes Frege's logicism.

His logicism is said to be a universal language in that this will bypass all the barriers of language which are present in our world. As already mentioned, the present of the multi-languages on earth is an obstacle for communication and traction. To understand the other who

[269] Victor Howard Dudman, "From Boole to Frege" in *studies on Frege I logic and philosophy of Mathematics*, Matthias Schirn, Problemata Frommann-Holzbrog 42, 1976, p. 137.
[270] Steeven Chapados, *Op.cit.*, p. 218.
[271] Gottlob Frege, *Idéographie*, p. 6.

does not speak my language, I need to take time to study his own language and vice-versa. Or I will need a translator to help me translate what he or she is saying even thought I will not be sure if the translator is not sheeting me. And if I have more than one customer speaking each his own language will I also called the number of translators? All these renders communication and businesses exchange to be low. Also, the traveling abroad is a difficulty due to these multi-languages. People face difficulties in going abroad because of the present of another language which they have to study before going there.

The only way for us to overcome such a problem is to invent a universal language that will not take into consideration any speaking language. A language that can be learn and understand by each and everyone in the world. Such a language is what Frege called a formula language based on arithmetic. The used of arithmetical notations in language will permit us to have a language that does not change according to time and space. The transaction will be free, communication will be easier, and transport will be free. No need to study a language to understand each other or a translator to interpret what the other is saying and spending money for that, the only thing to do is to learn the formula language.

8.2.2. The elaboration of a new language: ideography.

Ideography as defined Frege: "*is a set of rules by means of which we could directly express thoughts in written or printed symbols without the intervention of spoken language*"[272] This means that, the Begriffsschrift is a new language based on arithmetical formula which aims at the expression of contents by means of the written signs without passing through spoken language. According to the author of "Sense and references", our signs renders present to what is absent, invisible and inaccessible to the senses: signs, think Frege, are indispensable for thought as the microscope to the doctor. Without the signs, we would hardly elevate conceptual thinking. In fact, to Frege, science is ambiguity and to free it from this infirmity, we must submit it to a language of signs not audible which cannot prevent the interpretation errors and prevent the fallacy of reasoning but to written signs. The use of signs in the new language of Frege are very important. Frege thinks that it is therefore highly important to devise a mathematical language that combines the most rigorous accuracy with the greatest possible brevity. He affirms in this case that:

[272] Gottlob Frege, this is our translation of *« un ensemble de règles au moyen de laquelle nous pourrions directement exprimer la pensée dans des symboles écrits ou imprimer sans intervention de la langue parlée* », in *Écrits logiques et philosophiques*, p. 69.

> *the signs have, for thought, the same importance as navigating, the idea of using the wind has to go against the wind. May no one despise signs, both depend on their relevant choice! And their value is not diminished if after a long use it is no longer necessary to produce the sign, if we no longer need to speak aloud to think. We think no less in words and, if not in words, in mathematical signs, or in otherwise.*[273]

In other words, spoken language are defective, allusive and does not satisfy the univocity of science; it leaved allusively emerge, not only confusion of significations, but also, variations of non-equivalent words. Among the several example, we can quote the word "hors" which can designate an individual but also a species, as in the proposition "a horse is an herbivore" and this word at the end can have a concept sense, as in the proposition "this is a horse". To Frege, ordinary language is not governed by logical laws which can guarantee the formal rigorous of thought. In clear, by its nature, the spoken language does not offers a good logical guaranty. While the sign language or written language are indispensable for science. The written word has a more duration than the spoken word. With the written words, we can over run several thoughts without been afraid that this will be changed and verified as possible.

8.2.3. Conceptography: short and easier way of writing system.

Writing refers to any system of communication based on conventional, permanent, and visible signs. A writing system is a set of rules relating a given set of written signs of the linguistic units represented. The set of physical writing signs is called a script. Writing system may differ from one another in the script used, in the underlying set of rules, or in both. This may be divided in two according to what kind of linguistic unit their signs represent: the logographic writing, each sign represents a meaningful element like a word or a morpheme. The phonographic writing, each sign represents a phonetic or phonological element with no reference meaning. Most modern writing systems are phonographic. The phonographic writing system may be syllabic or phonemic according to whether each sign represents a syllable or a phonic. Very broadly speaking, therefore, there are three basic types of writing: logographic, syllabic and phonemic. One of the examples of the logographic writing is the Chinese writing which is the only full-fledged logographic system in current use. One of the standard examples of syllabic writing is Japanese Kana. The Latin alphabet, as well as other alphabet derived from Greek, is design as a phonemic writing. English language is an example of phonemic writing.

[273] Ibid., p. 64.

Frege's Conceptography is considered as a solution to the most fundamental problem of ordinary language which is the long use of sentences to express our thoughts. In fact, our spoken languages are the use of long statement for communication. This fall under the phonemic types of writing. This takes time to be written and some time the phrase is not well constructed due to the several rules of this system of writing. The different rules of the phonemic writing include, grammar, vocabulary and conjugation and these most be respected in other to write a good and well-constructed sentence especially in the English language. These rules are not only applied to the English language but also to all the rest of the 7000 languages present in the world. The mastering of all these rules takes us time and the violation of one of these changes the sense of the sentence given it another meaning. Our languages are always made up of errors since there is no one on earth that could master all these rules and to write without errors.

The used of these long statements in our languages to express our thoughts can be reduce to a short symbolization today with the help of the use of the formula language proposes by the mathematician philosopher Gottlob Frege. The used of Frege formula language which is the system writing of our phonemic using arithmetical notations or symbols prevent us from long statements. Here there is no need of formulating a long statement, indeed we just need to know how to symbolize each statement using the different symbols of each word. For example, the following words can be symbolized simply as; negation (¬), implication (→), and (∧). Let us write the following phrases:

"**Paul is ill therefore he will not go to school today**." This long statement can be symbolized in a short way as**: P ∴ q**

"the only possibility to enter doctorate cycle is to have written a dissertation and defended." This statement can be written as: **P → (Q ∧ R)**

Let us symbolize the following argument:

No student is a researcher.

All leaners are researchers

Therefore, no leaner is a student.

This is symbolized as**: (∀*x*) (S(*x*)→¬R(*x*))**

(∀*x*) (L(*x*)→R(*x*))

∴ (∀*x*) (L(*x*)→¬S(*x*))

We can see from our examples that the formula language makes short and ease our expression of writing in the phonographic writings.

8.3. ON THE SOCIO-CULTURAL AND POLITICAL ASPECTS: THE PRINCIPLE OF CONTEXTUALITY.

It is important to note, Frege philosophy does not only end on his *Begriffsschrift* where his aim was to establish a formula language. But we also have one of his greatest books known as *The mathematical foundation* where he developed a new approach of the philosophy of language called the principle of contextuality. The principle of contextuality holds that a word has no meaning only in the context of the sentence see an assertion.[274] That is, "*only the propositions has a sense; only in the context of a proposition has a name meaning*"[275]This therefore means that there is no fixe definition for a given word as hold the Frege of the Ideography but that the definition of a word depend on space and time in which the word is been used Our purpose in this sub-chapter is to give the interest that such a conception may have today.

8.3.1. Interest of Frege contextuality in politics.

Politics can be conceived here as the art of ruling the city with all what goes with. To Sponville, politic "is all what concern the life of the city, and especially the ruling of conflic, the relation of force or power."[276] Such a definition live us to thing that politic is war or is just resume to the conquer and maintenance of power. Politics goes beyond this conception by been the normative science which treats of the organization of social goods. If this is the definition of politics, in what way can the priciple of contextuality be an interest to the ruling of the state and social affairs?

The importance of the principle of contextuality in politcs is its nature against all form of political authoritarism. In politics, the principle of contextuality put into an end all forms of political authoritatlism such as dictatorship and tyrany which is an obstacle to freedom. With such form of governance, the citizens are been restricted from their speech, everybody is

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[274] Philippe Nguemeta ; this is our translation of : « *un mot n'a de signification que dans le contexte d'une phrase voire d'une assertion* » in « Sur le conceptualisme épistémologique et la théorie de connaissance : Frege et Wittgenstein », *le journal de philosophie vol. 1 no 1*, 2022, p. 4.

[275], Ludwig Wittgenstein, *Tractatus Logico-philosophicus*, London, Kegan Paul, Trench, Trubner & CO. LTD, 1922, p. 34.

[276] André Comte-Sponville, this is our translation of : *« tous ce qui concerne la vie de la cite et spécialement la gestion des conflits, des rapports de force ou du pouvoir.* » in *Dictionnaire Philosophique*, Quadrige, PUF, 2001, p. 711.

summited to accept all the decisions been taken without no one contesting them feer to be given a sanction which can exceed up to death. There is no blossoming of the citizen.

In addition, the principle of contextuality open ways to the present of multi-parties which today lead us to what is known as democracy. As define Abraham Lincoln, democracy is the rulling of the people, by the people and for the people. This mean that, there is no more one person who takes decision and there is no more one person who speaks, but everybody is free to talk by given his or her own opinion for the well being of the nation. In this sense: "*the conformity of human view decides what is true and what is wrong? Is true and false what man say to being; and the agree in the language they use. It's not a compliance of openions, but a life form.*"[277] This mean that, a true nation can only be build on the concensus of many people who share ideas in a language they used, so as to bring solutions to the various problem that hinder the society. With such a philosophy, there is a plurality of ideas that are been assemble for the build of the nation.

8.3.2. The interest of Frege contextuality in the society.

As already said, we have to know that if Frege in his *Ideography* defend the idea of isomophism, which is the correspondent of a proposition with it object which he elaborated as the theory of sense and references, in the *Mathematical foundation*, our author will change his position by developing the principle of contextuality. As he affirm: "*we must clearly separate the psychologysm of the logic, the subjective of the objective. We must look for what words mean, not in isolation, but taken in their context.*"[278] In order words, the principle of contextuality put forward by Frege take into consideration that the sense of a concept and it's object depend on the contexts of emergence. The time ans space constitute the base for the interpretation of a given word.

The principle of contextuality is of great importance to our society in that it store cohesion and stability in the society. Most of human conflict usually degenerate from the differences in color, (where we have the white, the black, the red), the will for domination, marginalization, discrimination and difference in languages. Such conflict appears because of intolerance and the egoistic nature of man to previledge his or her own interests. This conflict

[277]Ludwig Wittgenstein, *Philosophical investigation*, Trans. En. G. E. M. Anscombe, P. M. S. Hacker & Jaochim Schulte, United State, Wiley Blackwell, 2009, p. 81.
[278] Gottlob Frege, quoted by Philippe Nguemeta, « Sur le conceptualisme épistémologique et la théorie de connaissance : Frege et Wittgenstein », *le journal de philosophie vol. 1 no 1*, 2022, p. 5.

some time lead to war which create social insecurity and instability in the community. With this, there is panick in the ididividuals. With the presence of the contextuality there is an opening to the others by tolerance which is the acceptation of the other in spite of the differences that can exist among them. This spirit of tolerance will create in the mind of all the individuals a love for the others and this will facilitate communication between them. Therefore, *« le langage n'est pas seulement un système de signes servant à communiquer des pensées ou à représenter le monde. Il est également et ceci au plus haut degré, une activité sociale ».*[279] This social activity of language creates intersubjectivity among the individual.

The principle of contextuality leads also relativity in the acceptation of the ideas of the others in the society as been a possible truth. The truth does not only come from an individual or decision from one person as the case in some married couple where the man is the only person to talk and the woman is there to obey. Such a conception of the society needs to be eradicated with the principle of contextuality where there is relativity and tolerance. Women also have a word to say in a couple because they are human beings. They have the same right as any other men. To Frege, language becomes a means for communication among individual.

8.3.3. The principle of contextuality in the cultural aspect.

By culture, we mean the sets of knowledges that a society transmit and valorize, especially that which carry on the past of the humanity: its history, believes and it works. This is the contrary of uncultured. In other words, culture is the different knowledges that are been kept and transmitted from one generation to another as the most precious thing that is common to that community. Each community share a given culture which is proper to the individual of that area. This culture can be resume: on dance, song, folklore, proverbs, myths. The presence of Frege contextuality favor the growth of multi-culturalism. Multi-culturalism are the presence of several cultures which are brought together. In fact, the present of multicultural fight against the theory of ethnocentrism which is the fact of putting our culture on top of the rest of the culture as been superior to them. This ethnocentrism creates marginalization, egoism, discrimination of the other cultures.

With the principle of contextuality, there is the equivocity or equality of all the cultures which exist. The is no superiority or inferiority of any culture. In fact, to speak like Senghor, there is cultural ecumenism. Ecumenism can be understood as an interconfessional movement

[279] Ludwig Wittgenstein, quoted by Clément Elisabeth et al., *La philosophie de A à Z*, Paris, Hatier, 2000, p. 252.

which promote common action between the different Christian courants, despite their differences in doctrines, with the aim of unity of Christians. In this case, cultural ecumenism in this interaction between the different to promote exchange. This is the universe of the given and receiving in which each people bring from their culture. In this point, been it in the cultural, linguistic, art, the principle of contextuality prevents our differences to be the source and conflict of different. The principle of complementarity: "*great the values of complementarity of Europe and the white man, and in fact all the rest of the other continent*."[280] This means that, the multiculturalism is a form of contextually which ensure a perfect and harmonious communion among the different cultures that are been present in our world. Frege's conception of contextuality are of so much importance to our society which bring the different cultures to the same level which promote the principle of tolerance, consideration, acceptation, unity, and equivocal.

Nevertheless, despite the present of the crises elaborated in chapter seven, this do not stop us from bringing out certain relevance which we think will be of most importance in the epistemological domain, socio-cultural and political domains. This constitutes the chapter eight. In the epistemological domain, we saw that the Fregean ideography is of interest in storing knowledge for a long run of time without it been changes, the precise understanding and translation of this knowledge, ant the invention of quantifications. The other is the linguistic interest the Fregean logic may have; the invention of a new system of writing, a universal language which ease communication and writing. The last was the principle of contextuality as an interest in the cultural, social and political aspect.

[280] Leopold Sédar Senghor, in Ruch, E., (ed), African philosophy, Rome, 1981, p. 226.

CHAPTER NINE: THE FREGEAN REVOLUTION OF LOGIC AND AFRICA TODAY.

After investigating on the challenges of Frege logical revolution in the epistemological, linguistic, socio-cultural and political domain, it is now question in this chapter to elaborate the Fregean revolution of logic in relation to the interest that this may have to the African continent. This means, that we will bring the different impacts that the ideography of Frege may have to help the African people in their way of thinking and in their development. Our investigation will be carryout on three aspects: Frege logic as a rejection of irrational and illogical knowledge. This will be the occasion for us to discourse irrational and illogical aspect of knowledge as dogmatism, mysticism and mythicism. The second will be the Fregean logic: away to analyze illusory and false propositions. Here, the occasion will be given to discourse on those false and illusory propositions used by politician, religious and lawyers. In the last aspect, this will be Frege logical revolution: a promotion of coherence and harmonious society.

9.1. FREGE LOGIC AS A REJECTION OF IRRATIONAL AND ILLOGICAL KNOWLEDGE.

9.1.1. Rejection of dogmatic knowledge.

Dogmatism is a form of thinking which is based on a truth been decisive, universal, immutable, and incontestable. The acceptation of this truth is determined by the obedient to a divine authority or human and not from a demonstration from its rational bases. This means that dogmatism refer to the intellectual attitude which is based on unshakable certainty and reject critics or doubt. This absent of auto-critic is accompanied by a reasoning which seem to be logic but found on partial a priori taken out of its context. Dogmatic knowledge become in this sense a presumptuous statement or such that lack a sufficiently rational and logical ground to explain things. Our religious believe and most of our cultural practices are dogmatic in that they do not accept doubt or criticism and uses believes in a divinity to justify their truth and actions. In any place where there is the absence of doubt or critic, there is intolerant, fanatism, sect and totalitarianism.

By putting on in place his logico-mathematic approach, Frege is excluding all those propositions that do have any logical and rational explanation. By logical and rational explanations, we mean those propositions which indeed of using the faculty of reasoning, which

permit us to judge between what is good or bad, what is illogical or logical (the four rules of though), uses "faith." Faith is the blind and absolute acceptant and belief on a person or divinity (gods or God), either due to an affirmation guarantee by a testimony or a sure document (Bible or Koran). For example, according to the Bible, the Christians believe that God is the creator of the universe by reading the book of the Geneses in chapter one: "*in the beginning, when God created universe, the earth was formless and desolate*"[281] This affirmation irrational and illogical in that; the question of who was there? Cannot be answer, who created God? at what time was this?

One of the most critics of Frege concerning this dogmatism is that; most of these persons or divinities do not have any reference. In this case they are non-referential and the thought loses value for us as soon we recognize that the reference of one of its parts is missing. For Frege, it is important that each name or word must have a reference or denotation. But now why do we want every proper name to have not only a sense, but also a reference? Why is the thought not enough for us? Because and to the extent that, we are concerned with its truth values. This is not always the case because some proper names lack reference, they only have a meaning or sense. For example, the word "God". The word is considered as an abstract word which has a sense but not a reference. There is nothing in which we can denote the word God in this our world. To Frege, if a word lacks a reference, it is said to be non-referential and, in this case, the proper name God is not true. That is why Frege affirms:

> *the fact we concern ourselves at all about the reference of a part of the sentence indicates that we generally recognize and expect a reference for the sentence itself. The thought loses value for us as soon as we recognize that the reference of one of its part is missing. We are therefore justified in not being satisfied with the sense of a sentence, and in inquiring also as to its reference*.[282]

From this assertion, Frege invite all the African people to renounce to all those dogmatic propositions on which most of the African people are dive in. African continent needs to get out of the cave of dogmatism which is an obstacle to their faculty of reasoning and carry out logical reasoning.

[281] The holy Bible, *Geneses*, chapter one, verse 1-2.
[282] Gottlob Frege, *Ecrits logiques et philosophique*s, p. 109.

9.1.2. Rejection of mystical explanation.

The word mystical refers to the believe that asserts itself in an individual or a party without trying to justify themselves through reasoning. This means that mystical is the used for esoteric, gnostic, theosophical types of knowledge which does not need verification process to justify it action. This has been used, too, for the whole area of psychic phenomena and occult happenings, borderland phenomena. In other word, mystical is the explanation and believes of super-natural or to paranormal phenomena actions which can be demonstrated or verified because this is above the physical and human reasoning. Any action that needs to be understood by human must be rationally and logically explainable through the human faculty of judging. With the present of mystical phenomena which we cannot deny the fact that they exist, are usually not capable of been explained by the human reasoning. The reason lies on that these phenomena do not follow any process which can be demonstrated and verified by all. This can only be possible if you are initiated. This therefore means that, the mystical phenomena can only be understood by some particular people who are of the domain. The knowledges of such phenomena are deserved for some special person who as we usually say "have four eyes" instead of having two like every other person. The additional two eye is what we name as the mystical eyes which can only be to those been initiated in the domain. Also, with these phenomena, nothing is caused naturally but all what is caused has a mystical origin even when we need only our reason to understand. Most of the African people even those who are said to be intellectual usually prefer to believe and accept this illogical and irrational explanation. That is why Ebenezer Njoh Mouelle wrote:

> *"the most afflicting show in underdevelopment is that of the irrationality in the behavior of man. In Douala, we rarely die of natural death and the disease itself does not come to us by the microbe, for example; it is necessarily the result of the malevolence of a third party. The heart attack is an unacceptable phenomenon; it is preferred by the explanation by the night lightning and occult unleashed by an uncle…*"[283]

The above affirmation of Ebenezer Njoh Mouelle shows the necessity for the African continent to start thinking by themselves and not believing on mystical explanation which are

[283] Ebenezer Njoh Mouelle, this is our translation of « *le spectacle le plus affligeant en situation de sous-développement c'est celui de l'irrationalité dans le comportement de l'homme. A Douala, on meurt rarement de mort naturelle et la maladie elle-même ne nous vient point par le microbe, par Example ; c'est nécessairement le résultat de la malveillance d'une tierce personne. La crise cardiaque est un phénomène inacceptable ; on lui préfère l'explication par la foudre nocturne et occulte déchainement par un oncle…* » In *De la médiocrité a l'excellence. Essai sur la signification humaine du développement*, Yaoundé, CLE, 2013, p. 11.

not logically and rationally founded. The solution to come out from this mystical explanation of phenomena is to embrace the logic of Frege which will help them to analyze all those false explanations of this phenomenon. Frege ideography becomes an imperative solution as for the black continent to get out of this. Frege thinks that: "*his Ideography can be successfully be used wherever special value must be placed on the validity of proofs, as for example when the foundations of the differential and integral calculus are established*."[284] This means that the Begriffsschrift of our author can be useful to the African people to established proof for a given proposition. This given propositions are those of mystical explanation which is still a mental illness to the African. So, the Ideography becomes the treatment which will help them to clearly differentiate what is illogical and irrational by applying the rules of the axioms and the different inferences been elaborated.

9.1.3. Rejection of mythical approach.

A myth is a fabulous story usually believe and taken into serious due to its popular origin and ancestral in which the impersonal agents mostly physical force, are represented in the form of personal beings, whose actions have a symbolic sense. This means that myths are the narration of imaginary ideas so as to represent a given reality or event. This imaginary story which have for long been considered as false is today a real reality which is at the origin of our thought. This can be seen with the African philosophy for whom their way of thinking is based on this imaginary which is a respond to the question of sense in their life. This therefore means that:

> *contrary to the idea received, the mythology is not reduced in a succession of tales and legends, more or less fantastic adventure stories, primarily for children. On the contrary, it represents a grandiose attempt to bring answers to the ancient question of the meaning of life, good life for mortals.*[285]

The definition above explained clearly that the idea of myth which was conceived as a fantastic or fiction narration of events with the aim of representing our real-life situation should not more been seen as useless. This may have some great attempt to bring a sense to our life in the physical and spiritual world. Moreover, this may be as the source for our knowledge been it epistemological in the sense of knowledge or in the sense of science. For example, it is

[284] Gottlob Frege, *Idéographie*, p. 08.
[285] Luc Ferry, *Mythologie et philosophie*, Le Figaro, Editions Plon, 2016, p. Préface.

believed that myth of the Odyssey was at the impulsion of philosophy. Our actor who is Ulysses has won the war usually called “the trojan war”.

To Frege, tales and legend of the myth violate the theory of sense and references. As already said, the theory of sense and references that which hold to all proper names or proposition, there must be a given donation of that sense. A name loses all its values when it violates this theory. In this case:

> *at any rate, one might expect that such sentences occur, just as there are parts of sentences having sense but no references. And sentences which contain proper names without references will of this kind. The sentence Ulysses was set ashore at Ithaca whiles sound asleep obviously has a sense. But since it is doubtful whether the name Ulysses occurring therein, has reference, it is also doubtful whether the whole sentence has one […]. In hearing an epic poem, for instance, apart from the euphony of the language we are interested only in the sense of the sentence and the images and feelings thereby aroused. […]. Hence it is a matter of no concern to us whether the name Ulysses for instance, has reference, so long as we accept the poem as a work of art*.[286]

The affirmation above is a demonstration that myths cannot be taken as true knowledge since most of the time, all the actors used in the allegory do not have a reference in which we denote the name. If the proper names used do not possess any reference, this therefore means that the whole proposition and eventually the whole narration is non-referential. Meaning that they do not exist because for something to have a reference is to have verification. The African people should stop thinking that the myth alone can be the source of solution to the present problem in which African is been facing. It is only with the help of logical and rational thinking that African can be able to think and bring solutions to their problems which is reduce to under-development been it mentally, physically or infrastructurally.

9.2. THE FREGEAN LOGIC: DECONSTRUCTION OF ILLUSORY AND FALSE PROPOSITIONS.

9.2.1. The use of sophistic in politics.

The term politic is a polysemous notion but we can say that from its etymology “*polis*” in Greek which refer to the normative science which treats of the organization of social goods. This is the branch of civics concerned with government and state affairs. In other words, politics recover all what has a link to the government of a community or state; the art of ruling, the

[286] Gottlob Frege, *Begriffsschrift*, pp. 62-63.

organization of power, the conduction of affairs, the various actions been provided, by an institution, organization, party, enterprises, or an individual. Politic concern all the various domains of a society and among these include: external relationship, internal organization and security, defend, publics finance, education, health, justice and culture. These domains, can be in the various field of action; inter-nation, national, regional, departmental, municipal. It is important to note that in a democratic state like our country Cameroon, the political actions are legitimate and legalized by the constitutional disposition. This mean that, all the state action been done are been voted there by been legitimate and legalize by the constitutional court.

In its evolution, the term politic had many reforms. In the antique period issue of politics was the question of the ideal regime. Many ideologists were been emerged in respond to this question; the Aristocracy, tyranny. This conception changes later with the income of Christianity in the medieval period. Politics was now oriented to the pope who had all the power given by the Supreme Being, God. Those who were ruling were the prince of the high class. This domination of politic which was reserve to the religious and prince changes in the contemporary period. This period came with the introduction of multi-parties which gave born to the democratization of institutions. It is this democratization of power that many countries in the example of Cameroon that are been engage in their politics.

The presents of these multi-parties have revived the sophistic, rhetoric and eristic language already fought by Aristotle through the syllogism and it laws. Due to the search and maintenance of power, politicians use the sophistic language for the captivation of the people attentions and participation for the elections. This is the language where by our main interest is to convince the listeners even if our argument has nothing to do with truth and ethics. So, the aim of the politician is not to tell the truth to the people but to convince them that what we say is the truth: "*the eristic dialectic is the art of carry out a debate in a manner that we always have reason, therefore whatever the means*."[287] Such a language is mostly used by the African politicians especially in Cameroon. This language is generally employed when the time of elections is already approaching, they start to make campaigns. Once the attentions of the people are been captivated and convinced and that the election are been won, that is where we notice that all what they were saying to us nothing is was been true. Because after the winning

[287] Arthur Schopenhauer, *L'art d'avoir toujours raison*, trad. fr. d'Helene Flora, collection « Librio », Paris, 2018, p. 7.

of the election, the politicians are now satisfying their own desire and those of their family members by forgetting all those promises made to the people.

According to Frege, such a language must be rejected because it is full of illusion and falsity. But one question still remained: how then can we analyze a false or illusory proposition of this politicians? Frege thinks that the only solution is to apply the laws of logic: "*logic is a science which is apply to what so ever, since the laws of logic are available for all disciplines, for all object. […] logic is an imperial and imperious thing which is impose everywhere, in any domain*."[288] The ideography is therefore important to analyze such positions.

9.2.2. The rejection of rhetoric in church.

A church is a community of believers mostly called Christians. A Christian community is made up of in one hand the individuals; the ecclesial members (in a hierarchy), sisters and brothers' congregations, assistance personnel's, and the faithful.) in the other hand, we have the infrastructures; the chapels, the bishopric, presbytery. There is the co-habitation of function in the Christians community: the institutional function which aim is the well-being of the personnel with the mission of the gestion of goods and man. We have the Christi function which is based on the dogma of Jesus Christ and his apostles, having for purposes the spiritual training and the awareness of the faithful. The principle mission of a Christian community is numerous, we have education, the awareness on morals rules, life, death, life after death, adoration, the knowledge of the good and bad, interaction in the society, justice, divine and so on. In the exercise of it function, the Christian community life and survival depend on the gift, fund collection, the various projects like agriculture, educational revenue, and pisciculture.

In its original structure, the Christian community is not that of sophists or politics. This means that, the purposes of the Christian community even though were for convincing by using the art of oratory, but not using eristic and sophistic. Its action of convincing was not for duping the Christians as does the sophists and politicians but this was based on the research of the good. The mission of the religious seems to converge the same as that of the sophist but the difference can be seen in their aim; the religious aims at adoration, ataraxy, spiritual virtue and life after death while the sophist aims is pleasure.

Today, with the incoming of the new Christian communities, it becomes very difficult to still separate these two activities. The reciprocity of the mission seems to be the eventual

[288] Gottlob Frege, *Idéographie*, p. 125.

share thing, the objective and the aims to be attaint are similar between the two chapel of activities. In our days, the religious communities are almost the same as the communities of sophists and politician: "*when considering the current arguments surrounding politics, rhetoric and religion, one is struck at once by how they seem to continue from one generation to another.*"[289] This means that the relationship of that has been link between religion, rhetoric and politics will never end. The three activities are now today mostly link to each other. They are not more separated in their objective and aims.

This can be seen with this new Christian community, where most of the pastors use sophistic argument to convince their faithful. African going to church is looking for blessing, deliverance, marriage, job, travelling abroad, and children. Through the use of these words, the pastors can manipulate his Christian through the use of oratory to convince them that God has answer to their prayer. In this case, the use of blessing water which is that of mineral water or mayor oil for the benediction and deliverance usually cost a huge amount of money (5000f-100,000f and above). This makes them to carry out a politics which will make their business to run. With the classical communities (catholic, protestant and Muslim), the servant of God tied more and more conjuration with the politicians, sophists, pernicious sect. They abuse of their faithful and special discourse to put them, either in brotherhood, or in political parties.

Let us first note that the new rhetoric has been instructed in opposition to logical reasoning. Rhetoric, against the formal logic that seek to reason with the help of mechanic mean, fits into a reasoning whose various element are in solidarity. When a system like logic is applied to experience, it is necessary that it is re-inserted into our belief. This makes an appreciable difference between demonstrative reasoning (formal logic) and argumentative reasoning (rhetoric). In formal language, the signs are supposed to be ambiguity and organized inside formalized rules; while the argumentative reasoning, it is in the natural language. The demonstrative reasoning is based on axioms; the argument does not seek to deduce from axioms its reasoning, but rather to provoke and increase the accession of an audience. So, it never happens out of context. This ambiguity found in natural language is inherent in the affectation argument, because the choice is not only depending on the interpretation, but also the presentation of certain aspects of the notions used.

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[289] Brett Lunceford, "Rhetoric and Religion in Contemporary politics", in *Journal of contemporary rhetoric*, vol. 2, No.2, 2012, p. 21.

In formal logic, the notion must be unambiguous and the term used to talk about it accurate. It is different in rhetoric where all kind of problems arise because of their use in natural language. The African therefore, and on more invited to use the Fregean logic to come out from the claw of these wolfs which aim is to enrich their selves.

9.2.3. Rejection of sophistic in courtyards.

The courtyard is the area or place where magistrate, lawyers, prosecutor, exert their functions, it is the place by excellent were justice is said. By justice, we refer to one of the four virtues; prudence, force, temperance and justice which respect the equality and legality. To Aristotle: "*is just what is in accordance with the law and what respect the legality; is unjust what is contrary to the law and lack legality*."[290] We usually distinguish three types of justice; the distributive justice which is concerned with the sharing of properties, good, services, merit and difficulty according to the necessity and the urgent in which we are face. For example, it will be for handicap person to school freely; the commutative justice which is that us in man exchange for service or good for either another service, good or money. This suppose there must be equality in the exchange; the corrective justice which is concerned with the instauration of prejudice, or problem. This is the most and common type of justice where rhetoric and sophistic languages are been used.

Today, in the African continent particularly in Cameroon, our prisons are full of prisoners not because they are all guilty but because most of our lawyer abuse of the language in rhetoric and sophistic to manipulate the consciousness of our prosecutor to defend the guilty and condemn the innocent. This means that, these lawyers are ready to demonstrate the for and contrary of a given thing just to protect their customers. Even the proof that are been present for examination have nothing to do with the truth but he is capable of demonstrating that the present proofs are true from his language of eristic. The reason of this, is because the lawyers in Cameroon do not have a fix revenue but their revenue depend on the numbers of process been won or lost from their customers.

It becomes therefore important, for our lawyers in dependent of their grade to study logic. Not only the traditional logic but also and mostly the mathematical logic which will help them to analyze all the false and illusory propositions been established by those partisans of rhetoric and sophistic. The African states must ensure that in the training of the lawyers they

[290] Aristotle, *Nicomachean Ethics*, trans. en. W. D. Ross, Kitchener, Batohe Books, Book V 2, 1999, p. 74.

implemented the study of symbolic logic. This will enable us to reduce the number of innocent prisoners in our prisons. This will also enable us eradicate false and illusory propositions in our courtyard and the manipulation of prosecutor consciousness.

9.3. THE FREGEAN CONCEPTOGRAPHY: A SOCIAL AND CULTURAL INTEREST TO AFRICA.

9.3.1. The ideography as a social harmony and stability for African.

The most important aim of every individuals living in a given community is that of having a social cohesion and stability. By social cohesion, we refer to the extent of connectedness and solidarity among groups in a society. This is the social process which aims to consolidate plurality of citizenship by reducing conflict of inequality, discrimination, marginalization, socioeconomic disparities and fractures in the society. Besides, stability here refers to the state where there is no movement of conflict among individuals in a community. This means that this is a situation where people live in peace without any dispute or war.

When we look at the African continent today, it is very difficult to talk of a social cohesion and stability. Most of the African countries are in serious conflicts which create instability and insecurity. These conflicts can be external, this means among the two or more countries. We have the case of Algeria and Mali. The other most known conflict is that of internal which is everyday increasing in almost all the countries in Africa. When we talk of internal conflict, we refer to those war been carrying in the country among the citizenship themselves. We have the case of Cameroon, the war in the North West and south west of the country. This war already lasting for more than six years in almost putting the country in an instability and insecurity.

When we search for the reason behind most of the conflicts been done internally in the different African countries, we will fine that this is due either to the present of bilingualism or the present of multi-ethnic groups which create confusion. This means that, most of the war present in these areas is because of languages. The language becomes the factor that creates conflict. We have on one part of those who believe that they are anglophone and other believe that they are francophone in that which contain the official language. In the ethnics, some hold that they are from this dialect and the others they are from this given dialect. Such a view creates marginalization, discrimination, authority, tribalism, and at the end lead to conflict. The presence of languages becomes an obstacle to social cohesion and stability.

To overcome such a language problem, the Fregean formula language becomes an adequate solution. With the formula language, there is no need for fighting again since this does not take into consideration the question of space and time. The language is universal, objective and translingual. By implementing such a language for communication will see their selves stepping one leg in front to social cohesion and stability.

9.3.2. The modelisation of the African culture.

Modelisation is the action of modeling a given thing. This means that, modelising refer to conceive, elaborate a model that permit us to understand, act, and attaint a precise aim. This is the adoption of a new form and way of doing thing which greatly differ from what was been done in the past or before. Modeling takes in this sense the modern way of representing, conceiving the African culture. By African culture, we refer to the different values that black people valorize and transmit from generations to generations. These values include in their selves; dances, norms, arts, proverbs, myths, folklore, language and songs. All these constitute the prop of the African identity. So, by modelisation, of the African cultures, we mean rendering these different aspects already given to take the part of the modern world which is the world of numerisation and technolisation.

Most of the African cultures are still having difficulties in taking the parts if modenisation. The reason lies on the fact, the African are still having the idea of the restriction with the fear destroying what they have as originality. But today, with the increase in numerisation and technoscience which embrace in its selves the formulization of language, it becomes necessary and important for the African continent to modelise their cultures. Let us take the example of the African languages which we usually called the mother language or dialect. We will notice that most of the black communities are still having the oral way of expressing and transmitting knowledge or their values to their generations. Such a system faces many difficulties which make the knowledge been volatile with time. In addition to that, the language may be having some ambiguities which render it unprecise. This oral system of transmitting knowledge is volatile in that man is not a machine who has a fixe storing capacity which can by past the time and space. In addition to that, man has the nature of fallibility he is susceptible to errors. Due to this, we are not sure that, the knowledge that is been transmitted will keep it originality and exact meaning.

In this case, the modeling of the African language becomes of great necessity and importance. By modelising the African languages, this will facilitate the easy stored of

knowledge with certainty and exactitude without it losing any meaning or significations with time and space. They could now have a history in which they been critics because they are not having any text. Frege formula language can in this sense be use as an example in which they can based and found their new language. In this sense we can say that:

> *the initial postulate of all my company is that the scientific spirit is ready for the scientific modeling. This means that with inevitable abstraction and signification, it is possible to describe the human mind by a coherent system of calculable function. From this intuition, my research work consists of developing a model from the mind that meets the requirements of the contemporary scientific method and which exploits the data tank to the digital medium calculation power as much as possible*."[291]

From the above affirmation, our cultures especially our languages should be modelized in order to avoid misleading or confusion and to store our values for a given long period of time. It is time for the African to get up of their sleep and for to look for a better mode which can help them to develop not only their countries but also their culture.

[291] Pierre Levy quoted by Marguerite Ngo Nkot Penda, *Langage naturel et automatisation de la pensée. Une analyse de la sphère sémantique I : computation, cognition, économie de l'information de Pierre Levy*, dissertation in philosophy, supervised by Pr. Thomas Minkoulou, University of Yaoundé 1, 2022, pp. 78-79.

GENERAL CONCLUSION

Reached at the end of this dissertation, which had as theme: **Frege and the logical revolution: a reading of the Begriffsschrift**, we seek to solve the problem of the epistemological pertinence of the Fregean logical revolution and innovation. In other words, we want to investigate at how the mathematician philosopher Gottlob Frege through his formula language has greatly influence and change the field of logic. To well conduct this our reflection, we have elaborated three main questions: what are the pre-Fregean foundation of logic? Of what relevance can be the ideography that Frege proposes to substitute the traditional logic? Is this logico-mathematical language sufficient enough to disambiguate the errors of ordinary language? In final, what are the various philosophical interests that this might have? The analytico-critics has been the method on which all our work has been carrying on.

From this method, we constructed our work into three great points which constitute our three parts; each having three chapters.

Our first part was articulated on the Pre-Fregean foundation of logic, where we gave the context of emergence of Frege philosophy. Starting with the founder of formal logic, Aristotle in chapter one, we elaborated all the traditional logic of this author. We saw that Aristotle is considered as the father and founder of formal logic which is a logic based on rules that guide the validity of an argument. Aristotle aim of elaborating such rule was to reject all the false and illusory arguments elaborated by the sophists of his time. The sophists were abusing of language to convince their listeners even if in their arguments nothing was been true. Aristotle system of logic is based on three mains points: the classification of propositions which are four of them, the A, E, I, O. The A universal affirmative, the E universal negative, the I particular affirmative and the O particular negative. The second point is that of the inferences where we have the immediate inference which is the square of opposition and the mediate inference which is the eduction. The last and most important point is the syllogism with its four figures, laws and modes. Still in this chapter, after elaborating the Aristotelian system of logic, we gave some of its limits: the tautological aspect, polysemy aspect.

In the chapter two, which concerned the modern classical logic and its problem, we came out with those philosophers of the modern period who elaborated a theory on logic. We look at three of these philosophers: the first was Descartes with the rejection of traditional logic in favor of what he called the four rules of the method as a way to search truth in science. The second was the Port Royal logic with its notions of ideas, the nature of judgment, on reasoning and on the method which they developed in their book known as *Logic or the art of thinking*.

In this book, Arnaud and Peter elaborated a logic of term and taken back most of the Aristotelian logic mostly in their part concerning reasoning. The last is Kantian heritage of logic with his book on *Logic*.

The chapter three concerned the Leibniz, Boole and mathematical logic where the tentative for a formula language started with the work of Leibniz. In his universal calculus, Leibniz was already thinking of how to create a new language based on mathematics. He is not mostly viewed as a one of the founders of this logic because he did not write a book. Most of all what we know of its logic comes from some fragments been collected and constituted as his logic. It is Boole that most the history of logic considered to be the founder of the mathematical logic which will later be taken and systematize in a rigorous way by Frege. It is this systematization of Frege formula language which lead to the second part of our work.

The second part as already enunciated above is concerned with the Fregean revolution of logic where we bring out into detail the ideography of Frege consider as his formula language. But before investigating in the ideography, it is important that we first analyzed the critics been address by Frege concerning the Pre-Fregean logic. These weaknesses constitute the chapter four. One of the critics been done is the question of the judgment where Frege rejects the notion of subject and predicate, the types of judgment and the rejection of particular and universal judgments. Frege also reject the natural languages. Meaning that, Frege rejects the ordinary languages. To him, ordinary languages are ambiguous and render language confusion in communication.

The chapter five which is articulated in the philosophical foundation of Frege Conceptography. Here, we analyses the mains symbols that constitute Fregean ideography, the rules and the axioms of the Ideography. Concerning the rules, we have more than twelve rules and the axioms are of nine divided into four: three conditionals, three negations, two identities and one generality. In this same chapter, there is the demarcation between the two most founder of logic: Aristotle and Frege. This can be seen in the methodological and the technical plan.

The chapter six, is concerned with the Frege system of logic where we fine thought and truth table, the square of opposition. Apart from this, there is the present of the influence of Frege philosophy to some of his contemporary. We have the appropriation of the Vienna Circle of the philosophy of Frege, the Wittgenstein appropriation of Frege logic. With the later there is the alikeness and divergence.

In our chapter seven, our main preoccupation has been that of elaborating the different crises hinders by the Fregean logical revolution to be establish. The first difficulty encountered by Frege was the hinderance in founding a formula language due to the present of multi-languages which are about seven thousand in the world, the mutation of this language according to time and space and the present of undecidable proposition which cannot be proven nor represented. The second is that Frege is considered as an anti-evolutionist which can be seen with the implementation if fixe symbols in language making the language to be fixed without any mutation meanwhile language has to be dynamic. We have the rejection of any evolutionism in logic. To him, the laws of logic have always been the same. The third is that of the mastering of this ideographic symbol which takes time to be master and to retain all the laws going with them. This means the notations and signs present in the ideography necessitate one to have an opening mind and to like mathematics.

Our chapter eight is constituted of the relevance of Frege logical revolutions. In the epistemological domain, we saw that the Fregean ideography is of interest in storing knowledge for a long run of time without it been changes, the precise understanding and translation of this knowledge, ant the invention of quantifications. The other is the linguistic interest the Fregean logic may have; the invention a new system of writing, a universal language which ease communication and writing. The last was the principle of contextuality as an interest in the cultural, social and political aspect.

Finally, in the last chapter, we attempt to give the relevance's that Frege formula language may have in the African continent. We started by looking at how this can be a useful tool to eradicate all the irrational and illogical arguments been hold by some Africans. This include the explanation of things using mysticism, mythic, and witchcraft. African should abandon all the illogical and irrational approach of explaining phenomenon. The logico-mathematic of frege in this sense is a specific tool. Another is the rejection of illusory and false propositions used by the man of God, the politicians and the lawyers. Most of these people cited usually used sophistic and rhetoric argument to persuade their listeners. For the man of God, the sophistic argument been hold is to convince the Christians even if in his argument there is nothing true. This is the same thing done by the politicians who tries to convince his listeners during the time of campaign by promoting them many things in exchange of their votes. This is also the same thing done by the lawyer to convince the persecutor of the innocent of the guilty and sending the innocent in prison as the guilty. It becomes imperative for the African to learn the formula language as not to fall in the drape of these people. The last we show that, for the

African continent it is important to learn the formula language so as to also formulize their own language by been sure that this last for a long run.

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