

Stress-strain behaviour of bamboo beyond its elastic limit under uniaxial compressive load, using the principles of plasticity and damage mechanics

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Abstract

*The experimental determination of the behaviour of bamboo under the action of monotonic uniaxial compressive loading and cyclic uniaxial compressive loading, as well as the subsequent formulation of mathematical models to study this behaviour are presented in this work. Several samples of the bamboo species *Oxytenantera abyssinica*, from the Congo Basin Rain Forest were tested to collect the necessary data for this study.*

Under the loading regimes applied, it was found that the behaviour of bamboo above its elastic limit is non-linear and this nonlinearity could be attributed to two distinct mechanical processes, which are plasticity and damage. These two degradation phenomena are best described by the theories of plasticity and continuum damage mechanics.

A mathematical model, known as Fozao-Foudjet 2, that can be used to study the behaviour of bamboo under the monotonic uniaxial compressive load, beyond its elastic limit is proposed. This model has been obtained by modifying Mazars and Pijaudier-Cabot model from the study of the behaviour of concrete under similar loading regime.

The major experimental parameters used to obtain the proposed model are computed from the tests results while others are obtained from a trial and error procedure.

The stress-strain curves produced from the proposed mathematical model are compared with the test results and the comparison shows that the results from the model provide a good agreement with the test data.

Keywords : *mathematical modeling, isotropic damage, bamboo, plastic strain, envelope curves*

Résumé

*La détermination expérimentale du comportement du bambou sous les charges de compression uni axiale ainsi que le développement d'un modèle mathématique pour étudier ce comportement sont présentés dans ces travaux de recherche. Plusieurs éprouvettes de bambou, *Oxytenantera abyssinica* du Bassin de Congo ont été testées sous des sollicitations monotones uni-axiales et cycliques uni-axiales pour collecter les données nécessaires pour effectuer cette étude.*

Dans cette étude, il a été trouvé que le comportement du bambou au-delà de sa limite élastique, sous l'action des charges de compression uni-axiale est non-linéaire. Cette non-linéarité peut être attribuée à deux processus mécaniques distincts. Il s'agit de la plasticité et de l'endommagement. Ces deux phénomènes de dégradation du bambou peuvent être mieux décrits par la théorie de la plasticité et celle de la mécanique d'endommagement des milieux continus.

Un modèle mathématique appelé Fozao-Foudjet 2 a été développé, et proposé pour étudier le comportement du bambou sous des charges de compression uni-axiale monotones. Ce modèle a été obtenu en modifiant le modèle de Mazars et Pijaudier-Cabot dans l'étude du comportement du béton sous un régime de chargement similaire.

Les paramètres expérimentaux essentiels utilisés pour le modèle proposé ont été obtenus à partir des résultats des essais de laboratoire alors que les autres paramètres ont été trouvés par ajustement numérique.

Les modèles proposés pour modéliser le comportement du bambou sous les charges de compression monotones et cycliques ont été comparés aux résultats des essais de laboratoire et cette comparaison a montré que le modèle proposé s'accorde très bien avec les résultats des essais.

Mots clés : *modélisation mathématique, endommagement isotrope, bambou, déformation plastique, courbes enveloppe*

1. Introduction

Bamboo is the most important non wood species that grows in most tropical and subtropical zones. It is a naturally occurring composite material found abundantly on the surface of the earth. It has been shown that bamboo is a superior alternate for manufactured wood composites. It is inexpensive, easily available, and has comparable physical and mechanical properties to wood (Chaowana P., 2013). Bamboo can be considered as a material that can be used in the construction industry to replace wood, because it has several advantages as compared to wood and other wood products. These advantages are fast growth rate, lightweight, high strength to weight ratio, low energy and simple processing techniques, production process is highly environmentally friendly and it is a low-cost material. It grows abundantly over the surface of the earth and it has been recognized as the fastest growing plant on earth, in addition to being a renewable natural resource (Ghavami and Hombeeck, 1981); Liese W., 1986; Ghavami and Culzoni, 1987; Ghavami 1988; 1995a; Ghavami and Solorzano 1995; Amada 1996, Ghavami and Rodrigues 2000 Ghavami et al., 2003). It is being recognized worldwide as a sustainable construction material, presenting good opportunities for environmentally friendly and sustainable resource supply (Chaowana P. et al., 2014). It is a raw material for numerous value-added products that can be applied as structural as well as non-structural elements of a building.

There are about 1200 to 1500 species of bamboo found growing in diverse climatic regions all over the world, from cold mountains to hot tropical regions. The different species can be grouped into three different types of root systems, which are the

sympodial (clumping), monopodial (running) and amphipodial (clumping and running) types of root systems.

Despite these advantages that bamboo has over wood, and the fact that it is a potential substitute for wood in the construction industry, it is not widely used as a construction material because very little information exists on its mechanical characteristics. This article is intended to present the mechanical behaviour of bamboo under uniaxial compression beyond its elastic limit. A constitutive model, that can be used to explain this behaviour and which was developed by Fozao Dennis and Foudjet Amos (Fozao et al., 2019) is presented in this article. The constitutive model is developed based on the coupling of the theory of plasticity and the principles of Continuum Damage Mechanics.

2. Material and methods

2.1. Materials

The materials used are *Bambusa vulgaris* and *Oxytenantera abyssinica*, two species of bamboo found in the Congo Basin Rain Forest, water, and cement. Several samples of the bamboo species *Oxytenantera abyssinica* were subjected to monotonic and cyclic uniaxial compressive loads in order to study their behaviour under these loading regimes. The sources of the materials, their geometrical characteristics and their moisture contents at the time of testing are presented. The specimens were tested at varied moisture contents, some of the samples were oven dried; others were soaked in water and others air dried. The geometrical characteristics and the moisture contents of the various samples tested are found on the tables 1 and 2.

Table 1: Geometrical characteristics of the *Oxytenantera abyssinica* specimens used

No	Specimen	Diameter (mm)		Length (mm)
		External (d2)	Internal (d1)	
01	T2 Bottom with node	39.50	19.50	188
02	T10 Bottom without node	37.00	16.50	185
08	T9 Bottom without node	40.05	34.30	193
09	T9 Top without node	38.00	32.90	190
10	T3 Bottom without node	32.40	25.15	189
11	T4 Bottom with node	34.25	24.65	184
12	T4 Middle with node	33.30	26.30	184
14	T1 middle without node	36.45	27.60	184

Table 2: Moisture contents of the specimens

No	Sample	Moisture content (%)	Observation
1	T2 Bottom with node	107.50	Samples soaked in water for 43 days before testing
2	T10 Bottom without node	87.50	
3	T9 Top without node	16.39	Samples air dried
4	T4 Bottom with node	17.14	
5	T1 Middle without node	15.98	
6	T4 Middle with node	15.07	
7	T3 Bottom without node	16,80	
8	T9 Bottom without node	15.05	Samples oven dried at a temperature of 103°C for 24 hours. The values of the moisture contents shown are values of air dried samples. The samples were tested at values of the moisture content closed to zero

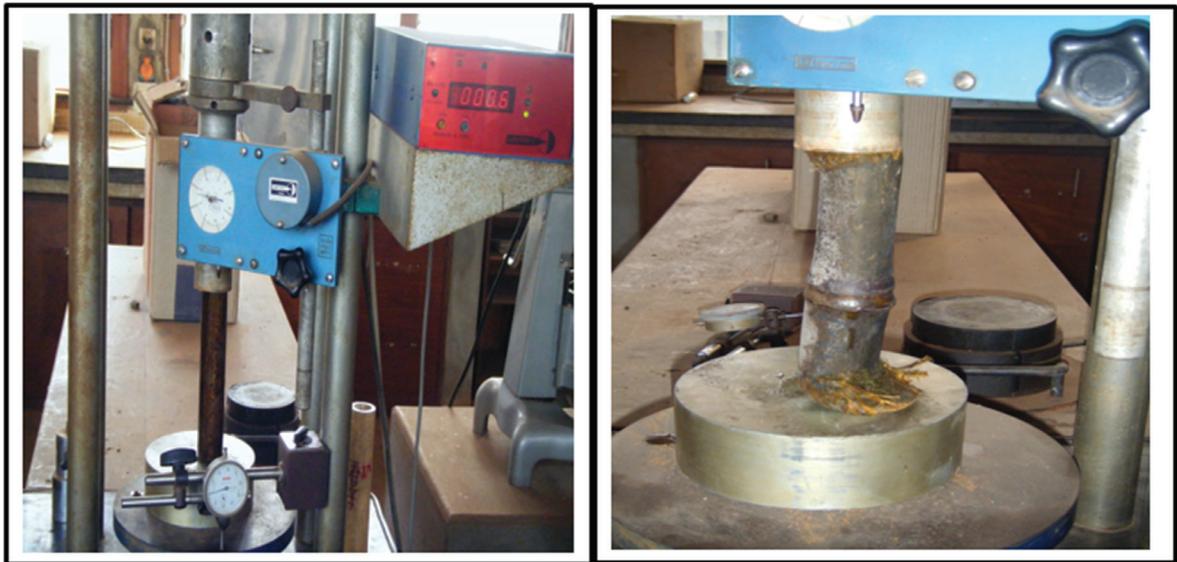


Photo 1: Specimens under the press for testing

2.2. Equipment used

The equipment used included : A hark saw, sand paper, a vernier caliper (manual version), a measuring tape, an electronic balance (the Scaltec mark), an oven (mark Heraeus), a bucket, the universal press measuring up to 2000kN, the CBR testing machine (LABOTEST) measuring up to 50kN equipped with a micrometer and a computer equipped with the Microsoft Excel software.

2.3. Experimental Program

After harvesting the bamboo culms, the growth buds were carefully trimmed for each species and the culms were divided into three portions, the lower, the middle and the upper parts. The specimens were

randomly selected and used for the experiments.

After preparing the bamboo samples, monotonic uniaxial and cyclic uniaxial compressive tests were carried out on them. The materials were tested to failure. For the cyclic tests, several cycles of unloading and reloading were performed on the materials and it was realized that most samples underwent several cycles before complete failure. For the monotonic loading, the samples were loaded continuously until complete failure of the bamboo sample. The deformations were measured using the micrometer attached to the testing machine, as the loads were applied. The test set up is as shown on photo 1 that follows, and this represents the setup of the CBR testing machine together with some samples under testing.

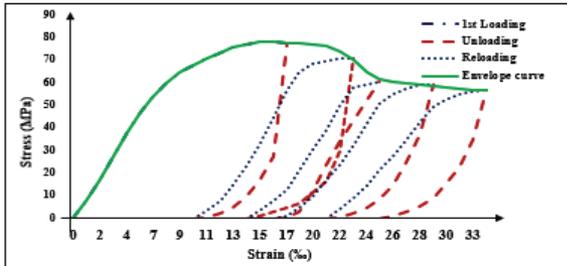


Figure 1a: T1 Middle without node, air dried sample

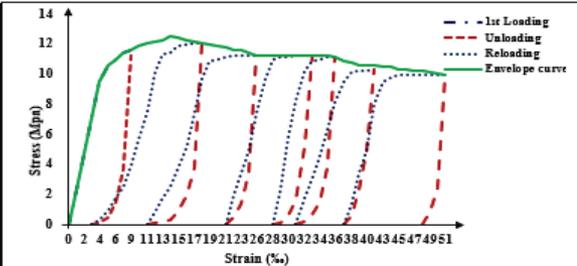


Figure 1b: T2 Bottom with node: Sample soaked in water containing some cement

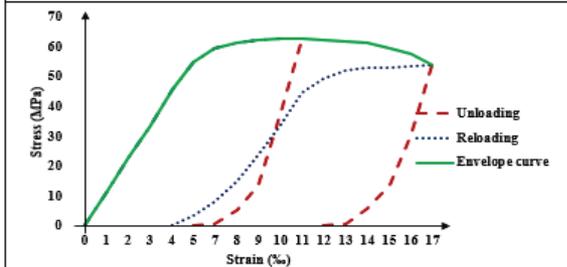


Figure 1c: T3 Bottom without node, air dried sample

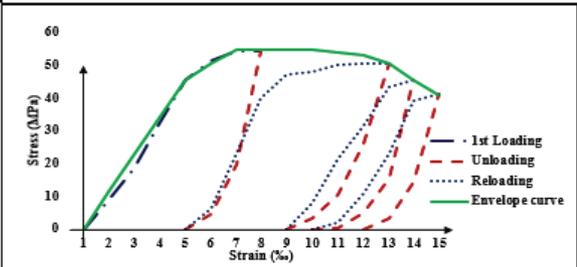


Figure 1d: T4 Bottom with node, air dried sample

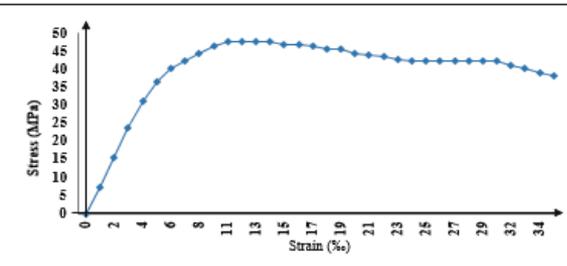


Figure 1e : T4 Middle Monotonic stress-strain diagram

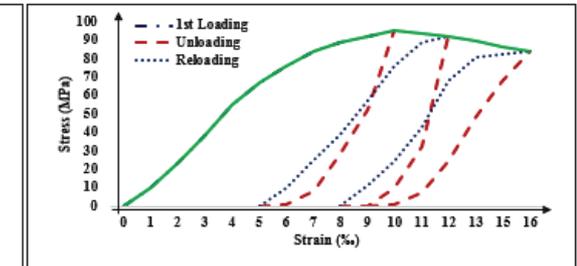


Figure 1f: T9 Bottom without node, oven dried sample

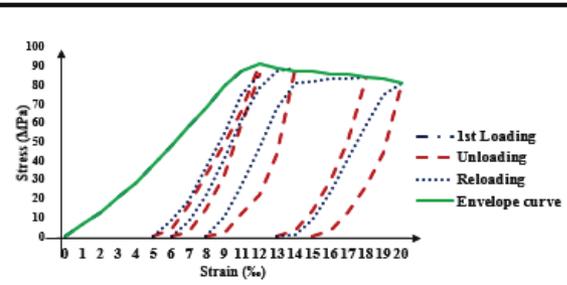


Figure 1g: T9 Top without node, air dried sample

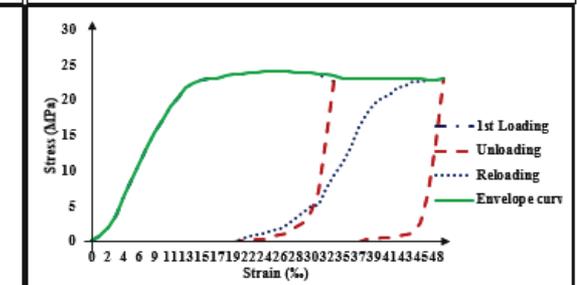


Figure 1h: T10 Bottom without node: Sample soaked in water

3. Results

The tests results are presented in the form of graphs showing the relations between the stresses and strains which are calculated using the original sample dimensions, from the loads applied and the deformations produced. Some of the graphs from the tests results are shown in figures 1a to 1h.

2.5. Nonlinear or inelastic behaviour of bamboo

From the curves plotted from the experimental data, it can be seen that the stress-strain diagrams

for bamboo under monotonic uniaxial compression present (Fozao et al., 2019) a linear elastic portion until its elastic limit is reached and after which a nonlinear or an inelastic portion exists right up to failure of the specimen.

This nonlinear portion is made up of a strain hardening portion which occurs after the elastic limit until the peak stress is reached thereafter there is a softening portion. The softening portion occurs after the peak stress until the failure stress.

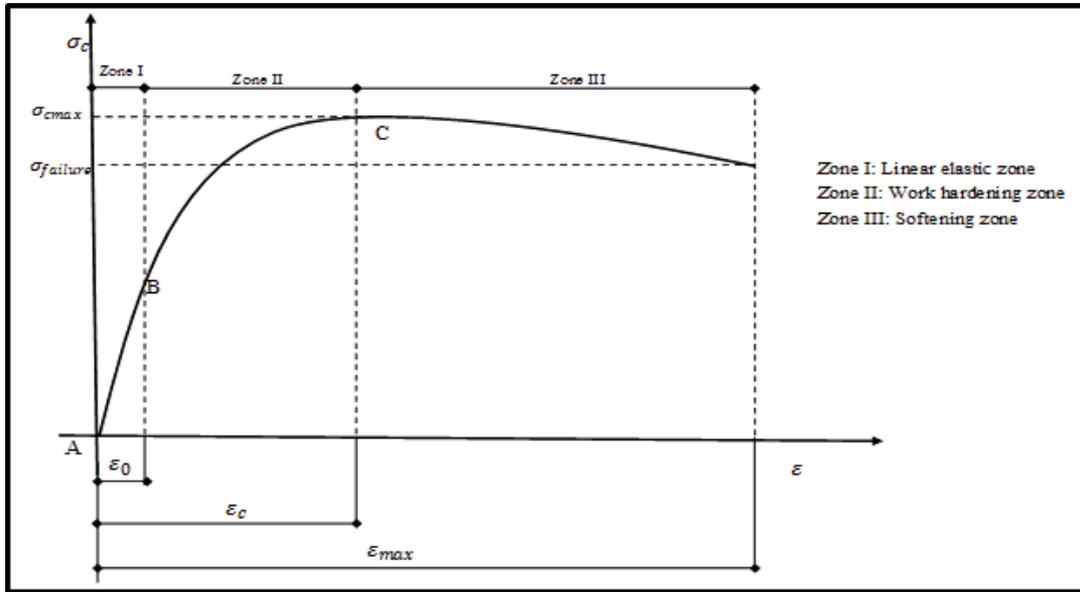


Figure 2: Typical monotonic uniaxial stress-strain curve for bamboo

From the curve in figure 2, it can be seen that the material exhibits almost a linear behaviour up to the proportional or elastic limit indicated by the point B. After this point, the material is progressively weakened by micro cracking up to the point C, corresponding to the peak stress. From the point B to the point C it can be seen that the stress increases with increasing strain, though at a reducing rate.

The zone between the points B and C corresponds to the zone of strain or work hardening. Beyond point C (that is in the post peak zone) the stress in the material gradually reduces as the strain increases. Therefore, the material is seen to exhibit strain softening effects, a state of the material where its stress decreases with an increase in the deformations leading to a reduction in the load carrying capacity of the material.

Beyond the elastic zone at the point B, the behaviour of bamboo under monotonic compressive loading is therefore highly nonlinear or inelastic. The total strain in the material is therefore made up of the recoverable elastic strain and the permanent plastic strain. If the material is unloaded beyond the point B, only the elastic strain can be recovered from the total strain.

Based on the studies carried out by other researchers on other engineering materials such as concrete, it is assumed in this work that initially the micro-cracks and micro-voids present in bamboo will not interact with the loading. However, as the loading increases, failure

mechanisms will occur where the micro-cracks and the micro-voids interact along localized zones of plasticity and damage. The localization can be attributed to a coupling of inelastic mechanisms such as micro-crack and micro-void growth with plastic flow. The initial defects in bamboo, along with any additional defects occurring during loading, will cause a nonlinear behaviour and a local weakness in the material.

These interactions will lead to a degradation of the global stiffness and to a subsequent decrease of the load carrying capacity of the material.

The nonlinearity of bamboo can be assumed to consist of both plastic slip and micro cracking of the material as the load gradually increases. Therefore, the theory of plasticity and the principles of continuum damage mechanics can be used to model the stress-strain behaviour of bamboo under uniaxial compressive loading (Fozao et al., 2019). Continuum damage mechanics offers principally more extended possibilities for crack analyses (Brekelmans et al., 1992 and Fozao et al., 2014).

3.1. Mathematical Modeling of the Behaviour of Bamboo under monotonic uniaxial compression

The stress-strain curves obtained from researches carried out on plain concrete under monotonic uniaxial compression show great similarities to those obtained for bamboo under similar loading regimes suggesting that expressions obtained for

concrete under these loadings can be modified and used to study the behaviour of bamboo. In this article a mathematical expression developed by Fozao and Foudjet named Fozao-Foudjet 2 is presented. This expression is developed by modifying the exponential stress-strain relationship proposed by Mazars and Pijaudier-Cabot (Sima et al., 2007), to study the behaviour of concrete under monotonic increasing uniaxial compressive loading.

The original expression from these researchers cannot be used exactly as they are given, to simulate the behaviour of bamboo under similar loading regime. This is because these expressions do not allow the softening branches to fit very well with the experimental results in certain cases. They actually under estimate the stresses in these cases.

The mathematical expressions found on the right of table 3 with the material constants, constitute the expression named Fozao-Foudjet 2. The expressions Fozao-Foudjet 2 is obtained by introducing the

Table 3: Comparison of the expressions from Pijaudier Cabot with expressions Fozao-Foudjet 2

N°	Original Expressions from Mazars and Pijaudier Cabot	Expressions Fozao-Foudjet 2
1	$\sigma = (1 - \varphi)E_0\varepsilon$	$\sigma = \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^\lambda (1 - \varphi)E_0\varepsilon$ with $0.0 \leq \lambda \leq 0.13$
2	$\varphi = 1 - \frac{\varepsilon_0}{\varepsilon}(1 - B) - Be^{\left(\frac{\varepsilon_0 - \varepsilon}{\varepsilon_c}\right)}$	$\varphi = 1 - \frac{\varepsilon_0}{\varepsilon}(1 - B) - Be^{\left(\frac{\varepsilon_0 - \varepsilon}{\varepsilon_c}\right)}$
3	$B = \frac{[\sigma_{c \max} - E_0\varepsilon_0]}{E_0 \left[\varepsilon_c e^{\left(\frac{\varepsilon_0 - \varepsilon_c}{\varepsilon_c}\right)} - \varepsilon_0 \right]}$	$B = \frac{[\sigma_{c \max} - E_0\varepsilon_0]}{E_0\varepsilon_0 \left[(k_0)^{\frac{\varepsilon_0 - \varepsilon_c}{\varepsilon_c}} - 1 \right]}$ with $k_0 = \frac{\varepsilon_c}{\varepsilon_0}$ and $1.00 \leq \lambda_0 \leq 1.38$

material constants κ_0 , λ_0 and λ_1 , to simulate the behaviour of bamboo under monotonic uniaxial compressive loading. The values of these parameters are given on table 4. The values of λ_0 and λ_1 vary as $1.00 \leq \lambda_0 \leq 1.38$ and $0.0 \leq \lambda_1 \leq 0.13$. When $\lambda_0 = 1.00$ and $\lambda_1 = 0.00$, the modified equations become exactly the same as those proposed by Mazars and Pijaudier Cabot to study the behavior of concrete. The material constant κ_0 is the ratio of the strain at maximum stress and the strain at the elastic limit. They can be used to study the behavior of bamboo under the monotonic uniaxial compressive loading regime. Table 3 shows the original expressions of Mazars and Pijaudier Cabot compared with the expressions Fozao-Foudjet 2.

3.2. Envelope Curves

From the test results, it has been realized that the envelope curves for bamboo culms subjected to cyclic uniaxial compression can be approximated by the monotonic stress-strain curves. The monotonic curve adopted as the envelope curve should verify some desirable characteristics such as:

- the slope at the origin should be equal to the initial modulus of deformation,
- it should describe correctly the ascending and the descending post peak (softening) branch and
- it should permit us to adjust the post peak behaviour to experimental results.

The mathematical model developed (Fozao-Foudjet 2), that is proposed to model the stress-strain behavior of bamboo under monotonic uniaxial compression can be used to model the envelope curves of bamboo under cyclic uniaxial compressive loading.

Table 4 : Parameters used in the models used to study the behaviour of bamboo

General					Mazar's Equation			Fozao-Foudjet 2		
Parameter Specimen	E ₀ (MPa)	σ _{c max} (MPa)	ε _c (x10 ⁻³)	ε ₀ (x10 ⁻³)	λ ₀	λ ₁	B	λ ₀	λ ₁	B
T1 Middle	8617.88	78.36	16.30	5.43	1.00	0.00	1.25	1.00	0.00	1.25
T2 Bottom	2231.41	12.52	13.83	3.191	1.00	0.00	0.508	1.38	0.13	0.206
T3 Bottom	10528.89	63.17	10.87	4.35	1.00	0.00	1.021	1.00	0.00	1.021
T4 Bottom	10513.40	54.82	8.70	3.26	1.00	0.00	1.40	1.00	0.00	1.40
T4 Middle	7419.67	47.60	13.68	4.21	1.00	0.00	0.84	1.126	0.125	0.84
T9 Top	8362.97	91.55	12.63	10.05	1.00	0.00	2.535	1.20	0.00	0.748
T9 Bottom	12875.48	95.01	10.36	5.18	1.00	0.00	1.99	1.00	0.00	1.99
T10 Bottom	1761.11	24.15	27.03	10.81	1.00	0.00	0.721	1.3	0.13	0.333

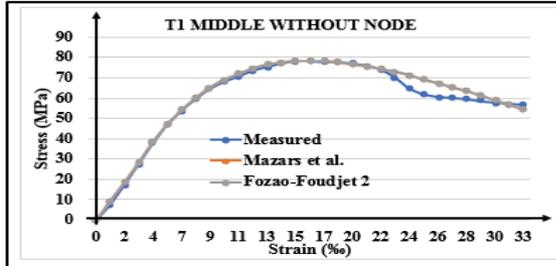


Figure 3a: Measured and calculated data compared

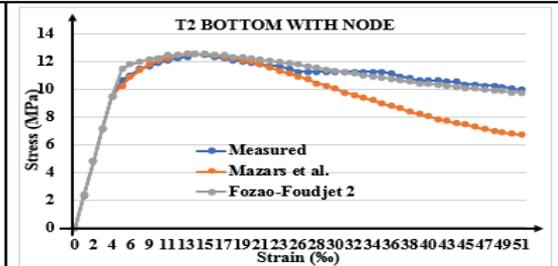


Figure 3b: Measured and calculated data compared

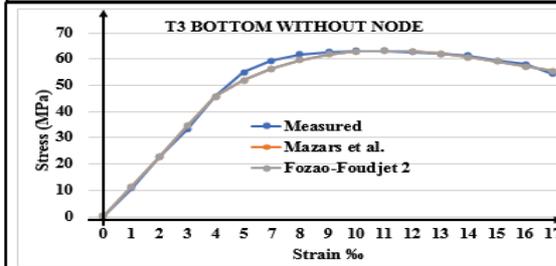


Figure 3c: Measured and calculated data compared

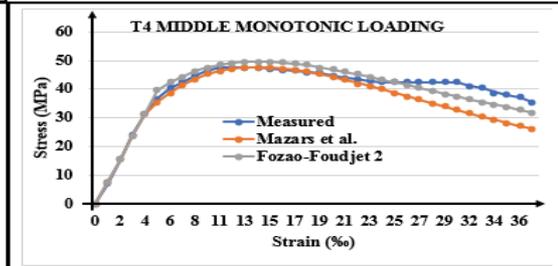


Figure 3d : Measured and calculated data compared

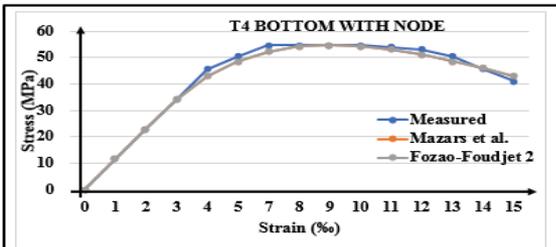


Figure 3e: Measured and calculated data compared

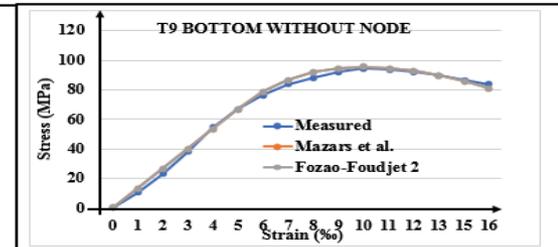


Figure 3f: Measured and calculated data compared

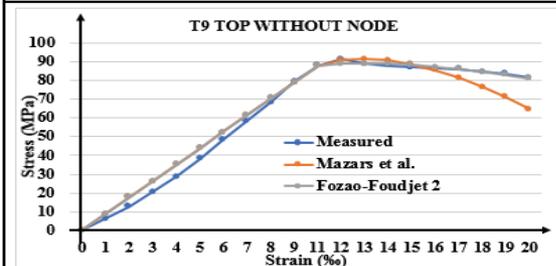


Figure 3g: Measured and calculated data compared

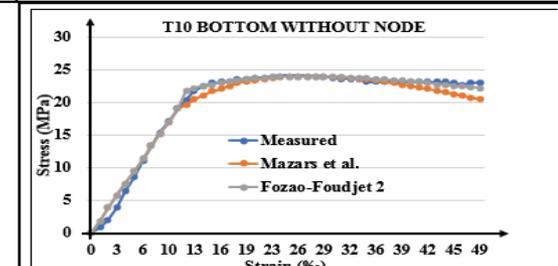


Figure 3h: Measured and calculated data compared

3.3 Presentation of the Results from the Models

The diagrams from figures 3a to 3h represent the stress-strain diagrams produced from the measured data and those obtained from the proposed models compared, for the envelope curves obtained from the cyclic and monotonic uniaxial compressive loading.

Each figure has three superposed graphs which are:

- Graphs from the experimental data (titled Measured).
- Graphs from the original mathematical expression

of Mazars and Pijaudier Cabot (titled Mazars et al.).

- Graphs from the mathematical expression Fozao-Foudjet 2.

The parameters that have been used in the models are found on table 4 that follows. This table contains parameters for the model Fozao-Foudjet 2 and those for the model Mazars and Pijaudier Cabot compared.

4. Discussion

Drying at very high temperatures and Soaking of Some Samples of Bamboo before Testing:

Some samples of bamboo were dried in an oven for 103°C while others were soaked in water containing some cement for 43 days.

Bamboo contains hygroscopic materials and can easily absorb moisture when placed in a humid environment. These materials may contain about 50-60% moisture content, depending on the felling season, area of growth of the bamboo and the bamboo species. The higher moisture content of bamboo allows it to rot, host fungi and be eaten by insects if it is not properly dried or cured. Bamboo will be very suitable for long term use only when it has been properly dried.

Therefore, drying of bamboo around temperatures as high as 103°C or more, will completely eliminate the free water in the bamboo material causing it to be dry enough to resist rot, fungi and insect attack. A minimum temperature of about 130°C is necessary to remove all the free water from the cell cavities of bamboo.

Soaking in Water

Soaking protects the bamboo from insects as it dries and leaches out starches that the insects would normally eat to survive. It also saturates the plant with water, evening out the moisture content, so as the bamboo dries, the water evaporates evenly and slowly, which can prevent cracking and splitting even in hotter weather.

Plastic Strain Behaviour of Bamboo

Permanent residual strains are produced in the bamboo material beyond the proportional limit after the loads are completely removed. Therefore, after the proportional limits, the behavior of bamboo deviates from the linear proportionality behaviour and becomes nonlinear. Bamboo contains a large number of micro cracks even before any load is applied to it. This property is very decisive for the mechanical behavior of the material. The micro-cracks may be caused by thermal expansion and shrinkage during temperature fluctuations.

The nonlinear behavior and the s-shape stress-strain curves of bamboo under uniaxial compressive stress can be associated with the micro-cracks propagation during load and stress-induced plastic flow in the specimen.

Comparison of Test Results with Proposed Models

From the stress-strain diagrams plotted above, it is

observed that the overall stress-strain behaviour of the proposed model Fozao-Foudjet 2 and tests results show similar configuration to each other as well as fit very well with each other.

Advantages and Inconveniences of the models Developed

The models developed and proposed to be used to study the behaviour of bamboo under monotonic uniaxial and cyclic uniaxial compressive loads possess some advantages and inconveniences.

Advantages of the model

The advantages of this model include;

- It is very easy to use.
- The results produced from it are very accurate and are the results expected.
- It can be used to model the behaviour of any species of bamboo under this kind of loading.
- Most of the required input for the model are obtained from the experimental data or graphs produced from the experimental data.
- It can easily be programmed using any computer programming language.
- The model does not need any yield surface or yield function.

Inconveniences of the model:

The major inconveniences of the model are;

- Many parameters are required to completely define it,
- Some of the parameters are determined by a trial and error method.

Perspectives for the Future

The future in the construction industry will be found in the use of bamboo and bamboo products for modern structures. If adequate research is carried out to improve on the quality of this material and its products, then it can be used very effectively as a good construction material. The following are some areas that have been suggested for further research.

- i. Bamboo fiber reinforced plastics and composites.
- ii. Characterization of the bamboo species found in the Congo Basin Rain Forest.
- iii. Behaviour of bamboo under various tensile loading regimes.
- iv. Behaviour of bamboo under tri-axial compression and other compression loading regimes.

- v. Behaviour of bamboo fibers under tensile loads.
- vi. Behaviour of bamboo poles under dynamic loadings.

4. Conclusion

Based on the present experimental investigations and the theoretical studies of the behaviour of bamboo under monotonic and cyclic uniaxial compressive loads, it can be concluded that the dominant failure mechanism observed for bamboo under uniaxial compression is axial splitting.

From the results presented, it has been realized that the mathematical model proposed can be used to study the behaviour of bamboo under monotonic uniaxial and the envelope curves from the cyclic uniaxial compressive loading. The results of the tests data agree very well with the mathematical model.

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